Unit 5

Geometry and Measurement

Focus

Use formulas to determine surface areas and volumes of three-dimensional shapes.

Problem Solving in Life Skills

Real-World Unit Project

Design That House Are you ready to become an architect? You've been selected to design and decorate a dream house. Along the way, you'll determine actual angle measures and wall lengths of the blueprint you'll be creating of your dream house. Your journey will begin soon, so pack your geometry tool kit.

Math Online Log on to glencoe.com to begin.

Geometry: Polygons

BIG Idea

- Identify and describe properties of two-dimensional figures.

Key Vocabulary

complementary angles (p. 514)

line of symmetry (p. 558)

similar figures (p. 540)

supplementary angles (p. 514)

🌐 Real-World Link

Tulips Holland, Michigan, hosts a Tulip Time Festival each May. Geometry is used to explain how a tulip shows rotational symmetry.
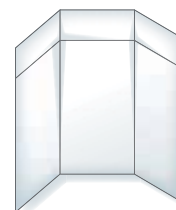
FOLDABLES Study Organizer

Geometry: Polygons Make this Foldable to help you organize your notes. Begin with a piece of 11" by 17" paper.

1 **Fold** a 2" tab along the long side of the paper.

2 **Unfold** the paper and fold in thirds widthwise.

3 **Open** and draw lines along the folds. Label the head of each column as shown. Label the front of the folded table with the chapter title.

What I Know About Polygons	What I Need to Know	What I've Learned

GET READY for Chapter 10

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICK Quiz

Multiply or divide. Round to the nearest hundredth if necessary.
(Prior Grade)

1. 360×0.85
2. $48 \div 191$
3. $24 \div 156$
4. 0.37×360
5. $33 \div 307$
6. 0.69×360

Solve each equation. (Lesson 3-2)

7. $122 + x + 14 = 180$
8. $45 + 139 + k + 17 = 360$

9. **SCHOOL** There are 180 school days at Lee Middle School. If school has been in session for 62 days and there are 13 days until winter break, how many school days are after the break? (Lesson 3-2)

Solve each proportion. (Lesson 6-5)

10. $\dfrac{4}{a} = \dfrac{3}{9}$
11. $\dfrac{7}{16} = \dfrac{h}{32}$
12. $\dfrac{5}{8} = \dfrac{15}{y}$
13. $\dfrac{t}{42} = \dfrac{6}{7}$

14. **READING** Sandra can read 28 pages of a novel in 45 minutes. At this rate, how many pages can she read in 135 minutes?
(Lesson 6-6)

QUICK Review

Example 1

Find 0.92×360.

$$
\begin{array}{r}
360 \\
\times\ 0.92 \quad \leftarrow \text{ two decimal places} \\
\hline
720 \\
+\ 32400 \\
\hline
331.20 \quad \leftarrow \text{ two decimal places}
\end{array}
$$

So, $0.92 \times 360 = 331.2$.

Example 2

Solve the equation.
$46 + 90 + p = 180$.

$$
\begin{array}{rll}
46 + 90 + p = & 180 & \text{Write the equation.} \\
136 + p = & 180 & \text{Add 46 and 90.} \\
-\ 136 \quad\quad -136 & & \text{Subtract 136 from} \\
p = & 44 & \text{each side.}
\end{array}
$$

The solution to the equation $46 + 90 + p = 180$ is $p = 44$.

Example 3

Solve the proportion $\dfrac{3}{8} = \dfrac{g}{48}$.

$\dfrac{3}{8} = \dfrac{g}{48}$ Write a proportion.

$\times 6$

$\dfrac{3}{8} = \dfrac{18}{48}$ Since $8 \times 6 = 48$, multiply 3 by 6 to find g.

$\times 6$

So, $g = 18$.

MAIN IDEA

Classify angles and identify vertical and adjacent angles.

New Vocabulary

angle
degrees
vertex
congruent angles
right angle
acute angle
obtuse angle
straight angle
vertical angles
adjacent angles

Math Online

glencoe.com

• Extra Examples
• Personal Tutor
• Self-Check Quiz
• Reading in the Content Area

▶ **GET READY** for the Lesson

ROLLER COASTERS The angles of descent of a roller coaster are shown.

1. The roller coaster at the right shows two angles of descent. Draw an angle between 44° and 70°.

2. Some roller coasters have an angle of descent that is 90°, known as a vertical angle of descent. Draw a vertical angle of descent.
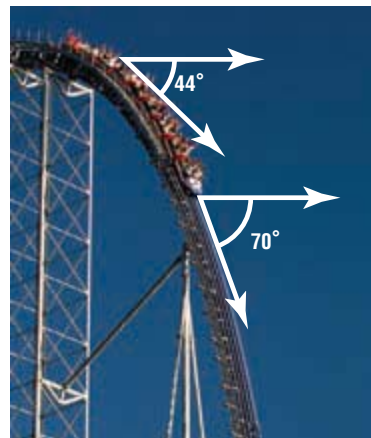
An **angle** has two sides that share a common endpoint and is measured in units called **degrees**. If a circle were divided into 360 equal-sized parts, each part would have an angle measure of 1 degree (1°).
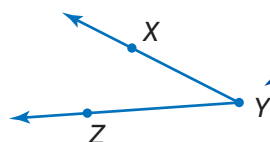
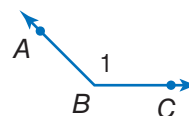A **vertex** is the point where the sides meet.

An angle can be named in several ways. The symbol for angle is ∠.

EXAMPLE Naming Angles

1 Name the angle at the right.

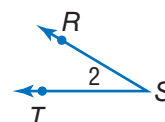• Use the vertex as the middle letter and a point from each side.
∠ABC or ∠CBA

• Use the vertex only.
∠B

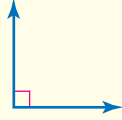• Use a number.
∠1

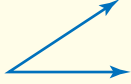The angle can be named in four ways: ∠ABC, ∠CBA, ∠B, or ∠1.

✓ **CHECK** Your Progress

a. Name the angle shown in four ways.

Angles are classified according to their measure. Two angles that have the same measure are said to be **congruent**.

Types of Angles

right angle   **acute angle**   **obtuse angle**   **straight angle**

exactly 90°   less than 90°   between 90° and 180°   exactly 180°

Study Tip

Right Angles The square corner indicates a right angle.

EXAMPLES **Classify Angles**

Classify each angle as *acute, obtuse, right,* or *straight.*

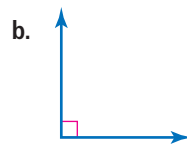**2**
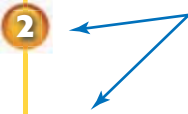
The angle is less than 90°, so it is an acute angle.

3

The angle is between 90° and 180°, so it is an obtuse angle.

CHECK Your Progress

Classify each angle as *acute, obtuse, right,* or *straight.*

b.   **c.**   **d.**

Study Tip

Classifying Angles You do not need to use a protractor to measure an angle in order to classify it as acute, obtuse, right, or straight.
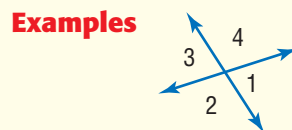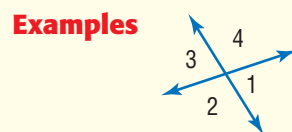
Vertical Angles

Key Concept

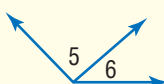**Words**   Two angles are **vertical** if they are opposite angles formed by the intersection of two lines.

Examples

∠1 and ∠3 are vertical angles.
∠2 and ∠4 are vertical angles.

Adjacent Angles

Words   Two angles are **adjacent** if they share a common vertex, a common side, and do not overlap.

Examples

Adjacent angle pairs are ∠1 and ∠2, ∠2 and ∠3, ∠3 and ∠4, and ∠4 and ∠1.

∠5 and ∠6 are adjacent angles.

Real-World EXAMPLE

4 **INTERSECTIONS** Identify a pair of vertical angles in the diagram at the right. Justify your response.
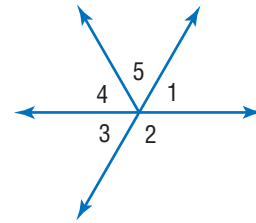
Since ∠2 and ∠4 are opposite angles formed by the intersection of two lines, they are vertical angles. Similarly, ∠1 and ∠3 are also vertical angles.

CHECK Your Progress

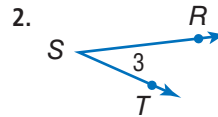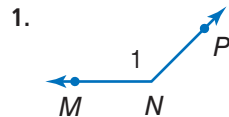Refer to the diagram at the right. Identify each of the following. Justify your response.

e. a pair of vertical angles

f. a pair of adjacent angles

CHECK Your Understanding

Examples 1–3
(pp. 510–511)

Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

1.

2.

Example 4
(p. 512)

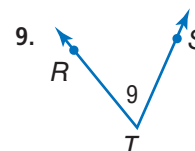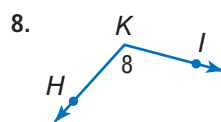**3.** **RAILROADS** Identify a pair of vertical angles on the railroad crossing sign. Justify your response.
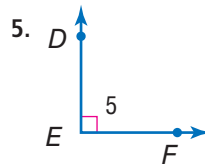
Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
4–9	1–3
10–17	4

Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

4.

5.

6.

**7.**

8.

9.

512 **Chapter 10** Geometry: Polygons
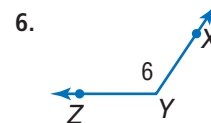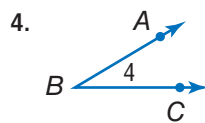
For Exercises 10–15, refer to the diagram at the right. Identify each angle pair as *adjacent*, *vertical*, or *neither*.

10. ∠2 and ∠5   **11.** ∠4 and ∠6   **12.** ∠3 and ∠4

13. ∠5 and ∠6   **14.** ∠1 and ∠3   **15.** ∠1 and ∠4
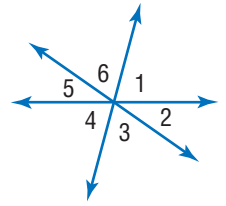
GEOGRAPHY For Exercises 16 and 17, use the diagram at the right and the following information.

The corner where the states of Utah, Arizona, New Mexico, and Colorado meet is called the Four Corners.
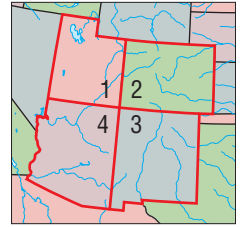
16. Identify a pair of vertical angles. Justify your response.

17. Identify a pair of adjacent angles. Justify your response.

H.O.T. Problems   **CHALLENGE** For Exercises 18 and 19, determine whether each statement is *true* or *false*. If the statement is true, provide a diagram to support it. If the statement is false, explain why.
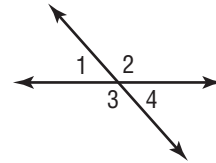
18. A pair of obtuse angles can also be vertical angles.

19. A pair of straight angles can also be adjacent angles.

20. **WRITING IN MATH** Describe the differences between vertical and adjacent angles.

TEST PRACTICE

21. Which word best describes the angle marked in the figure?

  A acute

  B obtuse

  C right

  D straight
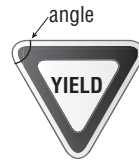
22. Which statement is true?

  F ∠1 and ∠4 are adjacent angles.

  G ∠2 and ∠3 are vertical angles.

  H ∠3 and ∠4 are vertical angles.

  J ∠2 and ∠3 are adjacent angles.

Spiral Review

A coin is tossed twice and a number cube is rolled. Find each probability. (Lesson 9-8)

23. P(2 heads and 6)   **24.** P(1 head, 1 tail, and a 3)   **25.** P(2 tails and *not* 4)

26. **PROBABILITY** A spinner is spun 20 times, and it lands on the color red 5 times. What is the experimental probability of *not* landing on red? (Lesson 9-7)

GET READY for the Next Lesson

ALGEBRA Solve each equation. Check your solution. (Lesson 3-2)

27. $44 + x = 90$ **28.** $117 + x = 180$ **29.** $90 = 36 + x$ **30.** $180 = 75 + x$

Complementary and Supplementary Angles

▷ MINI Lab

GEOMETRY Refer to ∠A shown at the right.

1. Classify it as *acute, right, obtuse,* or *straight.*

2. Copy the angle onto a piece of paper. Then draw a ray that separates the angle into two congruent angles. Label these angles ∠1 and ∠2.

3. What is $m\angle 1$ and $m\angle 2$?

4. What is the sum of $m\angle 1$ and $m\angle 2$?

5. Copy the original angle onto a piece of paper. Then draw a ray that separates the angle into two non-congruent angles. Label these angles ∠3 and ∠4.

6. What is true about the sum of $m\angle 3$ and $m\angle 4$?

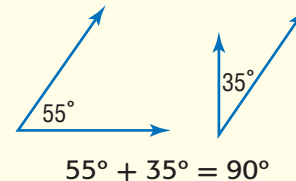7. Complete Exercises 1–6 for ∠B shown at the right.
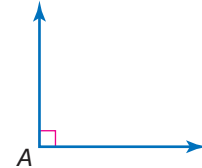
A special relationship exists between two angles whose sum is 90°. A special relationship also exists between two angles whose sum is 180°.

Complementary Angles	Key Concept

Words   Two angles are **complementary** if the sum of their measures is 90°.

Examples

$$m\angle 1 + m\angle 2 = 90° \qquad 55° + 35° = 90°$$
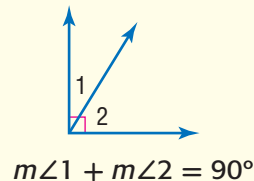
Supplementary Angles

Words   Two angles are **supplementary** if the sum of their measures is 180°.

Examples

$$m\angle 3 + m\angle 4 = 180° \qquad 40° + 140° = 180°$$
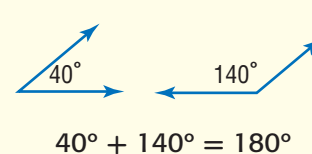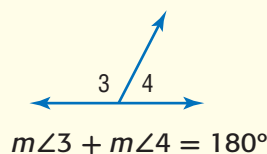
You can use these relationships to identify complementary and supplementary angles.

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.
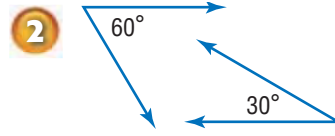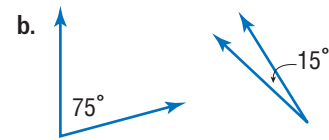
1
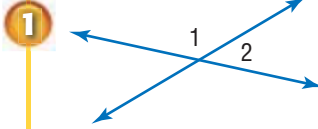
$\angle 1$ and $\angle 2$ form a straight angle. So, the angles are supplementary.

2

$60° + 30° = 90°$
The angles are complementary.

✓CHECK Your Progress

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.

a.

$85°$ | $90°$

b.

$75°$ $15°$
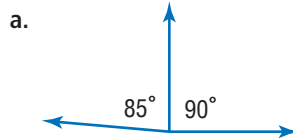
You can use angle relationships to find missing measures.

EXAMPLE Find a Missing Angle Measure

Reading Math

Perpendicular Lines or sides that meet to form right angles are perpendicular.

3 **ALGEBRA** Find the value of x.

Since the two angles form a right angle, they are complementary.
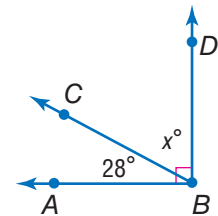
Words	The sum of the measures of $\angle ABC$ and $\angle CBD$	is	90°.
Variable	Let x represent the measure of $\angle CBD$.		
Equation	$28 + x$	=	90

$28 + x = 90$ Write the equation.
$\underline{-28 \qquad -28}$ Subtract 28 from each side.
$x = 62$

So, the value of x is 62.

✓CHECK Your Progress

c. **ALGEBRA** Find the value of x.

d. **ALGEBRA** If $\angle J$ and $\angle K$ are complementary and the measure of $\angle K$ is 65°, what is the measure of $\angle J$?

$46°$ $x°$
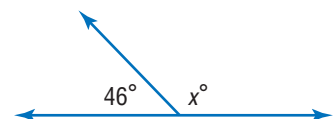
✓CHECK Your Understanding

Examples 1, 2
(p. 515)

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.

1.

135°

45°

2.

23°

67°
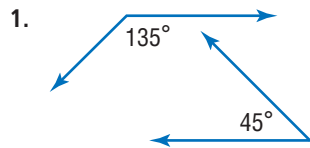
Example 3
(p. 515)

3. ALGEBRA Find the value of x.
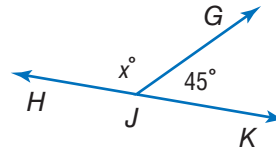
G

$x°$ 45°

H   J   K

▶ Practice and **Problem Solving**

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.

4.

43°

5.

61°   119°

6.

2

1

7.

1
2

8.

1
2

9.

1
2
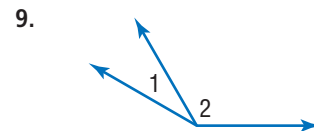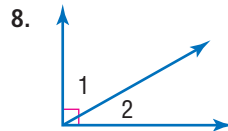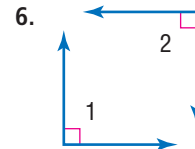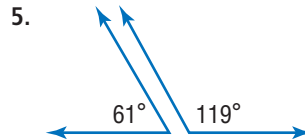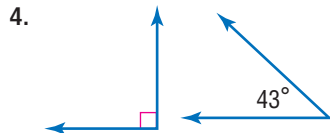
10. ALGEBRA If $\angle A$ and $\angle B$ are complementary and the measure of $\angle B$ is 67°, what is the measure of $\angle A$?
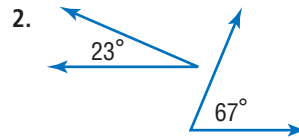
11. ALGEBRA What is the measure of $\angle J$ if $\angle J$ and $\angle K$ are supplementary and the measure of $\angle K$ is 115°?

12. SCHOOL SUPPLIES What is the measure of the angle given by the opening of the scissors, x?

$x°$
116°
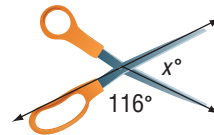
13. SKATEBOARDING A skateboard ramp forms a 43° angle as shown. Find the unknown angle.

43°

$x°$
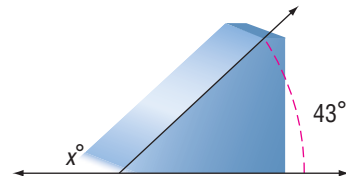
Use the figure at the right to name the following.

14. a pair of supplementary angles

15. a pair of complementary angles

16. a pair of vertical angles

B   C
A   K
      G   D
I   J
   H   F   E
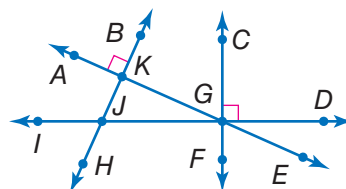
GEOMETRY For Exercises 17–20, use the figure at the right.

17. Are ∠1 and ∠2 vertical angles, adjacent angles, or neither? ∠2 and ∠3? ∠1 and ∠3?

18. Write an equation representing the sum of $m\angle 1$ and $m\angle 2$. Then write an equation representing the sum of $m\angle 2$ and $m\angle 3$.

19. Solve the equations you wrote in Exercise 18 for $m\angle 1$ and $m\angle 3$, respectively. What do you notice?

20. **MAKE A CONJECTURE** Use your answer to Exercise 19 to make a conjecture as to the relationship between vertical angles.
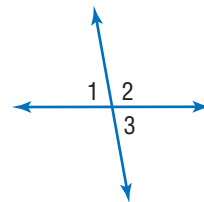
H.O.T. Problems

21. **CHALLENGE** Angles E and F are complementary. If $m\angle E = x - 10$ and $m\angle F = x + 2$, find the measure of each angle.

22. **WRITING IN MATH** Describe a strategy for determining whether two angles are *complementary*, *supplementary*, or *neither* without knowing or measuring each angle using a protractor.

TEST PRACTICE

23. In the figure below, $m\angle YXZ = 35°$ and $m\angle WXV = 40°$. What is $m\angle ZXW$?
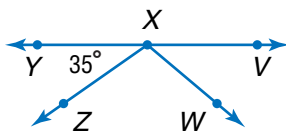
    A 180°   C 75°

    B 105°   D 15°

24. Which is a true statement about angles 1 and 2 shown below?
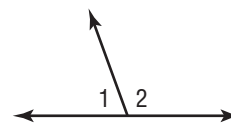
    F ∠1 is complementary to ∠2.

    G ∠1 and ∠2 are vertical angles.

   H ∠1 is supplementary to ∠2.

    J Both angles are obtuse.

Spiral Review

25. Name the angle at the right in four ways. Then classify it as *acute, right, obtuse,* or *straight.* (Lesson 10-1)

26. **MEASUREMENT** A house for sale has a rectangular lot with a length of 250 feet and a width of 120 feet. What is the area of the lot? (Lesson 3-6)
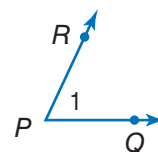
GET READY for the Next Lesson

PREREQUISITE SKILL Multiply or divide. Round to the nearest hundredth if necessary. (pp. 674 and 676)

27. $0.62 \cdot 360$ 28. $360 \cdot 0.25$ 29. $17 \div 146$ 30. $63 \div 199$

Statistics: Display Data in a Circle Graph

MAIN IDEA

Construct and interpret circle graphs.

New Vocabulary

circle graph

Math Online

glencoe.com

• Extra Examples
• Personal Tutor
• Self-Check Quiz

GET READY for the Lesson

VEGETABLES The students at Pine Ridge Middle School were asked to identify their favorite vegetable. The table shows the results of the survey.

1. Explain how you know that each student only selected one favorite vegetable.

2. If 400 students participated in the survey, how many students preferred carrots?

Favorite Vegetable	
Vegetable	**Percent**
Carrots	45%
Green Beans	23%
Peas	17%
Other	15%

A graph that shows data as parts of a whole is called a **circle graph**. In a circle graph, the percents add up to 100.

EXAMPLE   Display Data in a Circle Graph

1 **VEGETABLES** Display the data above in a circle graph.

• There are 360° in a circle. Find the degrees for each part.

45% of 360° = 0.45 · 360° or 162°

23% of 360° = 0.23 · 360° or 83°   Round to the nearest whole degree.

17% of 360° = 0.17 · 360° or about 61°

15% of 360° = 0.15 · 360° or about 54°

• Draw a circle with a radius as shown. Then use a protractor to draw the first angle, in this case 162°. Repeat this step for each section or *sector*.
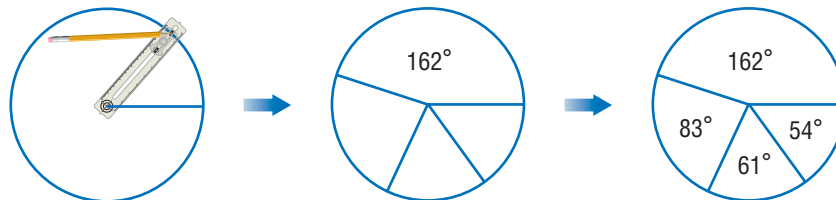
• Label each section of the graph with the category and percent it represents. Give the graph a title.

Check   The sum of the angle measures should equal to 360°.
162° + 83° + 61° + 54° = 360°

Favorite Vegetable
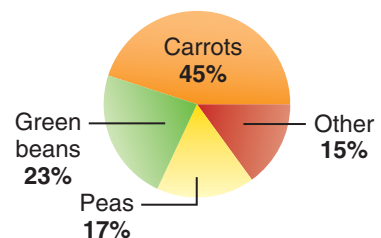
✔CHECK Your Progress

a. **SCIENCE** The table shows the present composition of Earth's atmosphere. Display the data in a circle graph.

Composition of Earth's Atmosphere	
Element	**Percent**
Nitrogen	78%
Oxygen	21%
Other gases	1%

When constructing a circle graph, you may need to first convert the data to ratios and decimals and then to degrees and percents.

EXAMPLE **Construct a Circle Graph**

2 **ANIMALS** The table shows endangered species in the United States. Make a circle graph of the data.

Species	Number of Species
Mammals	68
Birds	77
Reptiles	14
Amphibians	11

Source: U.S. Fish & Wildlife Service

- Find the total number of species:
  $68 + 77 + 14 + 11 = 170$.

- Find the ratio that compares each number with the total. Write the ratio as a decimal rounded to the nearest hundredth.

  mammals: $\frac{68}{170} \not\approx 0.40$ birds: $\frac{77}{170} \approx 0.45$

  reptiles: $\frac{14}{170} \approx 0.08$ amphibians: $\frac{11}{170} \approx 0.06$

- Find the number of degrees for each section of the graph.

  mammals: $0.40 \cdot 360° = 144°$

  birds: $0.45 \cdot 360° \approx 162°$

  reptiles: $0.08 \cdot 360° \approx 29°$

  amphibians: $0.06 \cdot 360° \approx 22°$

  Because of rounding, the sum of the degrees is 357°.

- Draw the circle graph.

  0.40 = 40%, 0.45 = 45%,
  0.08 = 8%, 0.06 = 6%
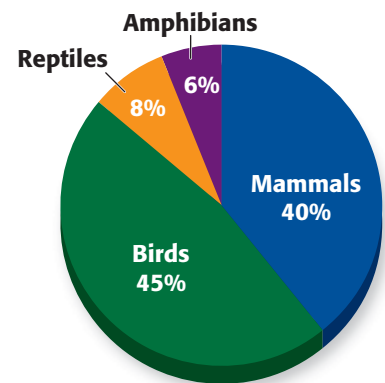
Check   After drawing the first two sections, you can measure the last section of a circle graph to verify that the angles have the correct measures.

Real-World Link . . . .

The Carolina Northern and Virginian Northern Flying Squirrel are both endangered. The northern flying squirrel is a small nocturnal gliding mammal that is about 10 to 12 inches in total length and weighs about 3–5 ounces.

Source: U.S. Fish and Wildlife Service

✔CHECK Your Progress

b. **OLYMPICS** The number of Winter Olympic medals won by the U.S. from 1924 to 2006 is shown in the table. Display the data in a circle graph.
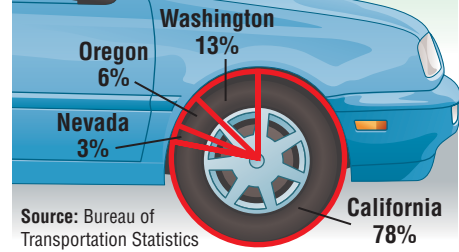
U.S. Winter Olympic Medals	
Type	**Number**
Gold	78
Silver	81
Bronze	59

EXAMPLES **Analyze a Circle Graph**

AUTOMOBILES The graph shows the percent of automobiles registered in the western United States in a recent year.

3 **Which state had the most registered automobiles?**

The largest section of the circle is the one representing California. So, California has the most registered automobiles.

U.S. Registered Automobiles in West

Washington 13%
Oregon 6%
Nevada 3%
California 78%

Source: Bureau of Transportation Statistics

4 **If 24.0 million automobiles were registered in these states, how many more automobiles were registered in California than Oregon?**

California: 78% of 24.0 million → 0.78×24.0, or 18.72 million

Oregon: 6% of 24.0 million   → 0.06×24.0, or 1.44 million

There were 18.72 million − 1.44 million, or 17.28 million more registered automobiles in California than in Oregon.

Study Tip

Check for Reasonableness
To check Example 4, you can estimate and solve the problem another way.

78% − 6% ≈ 70%
70% of 24 is 17

Since 17.28 is about 17, the answer is reasonable.

✓ **CHECK Your Progress**

c. Which state had the least number of registered automobiles? Explain.
d. What was the total number of registered automobiles in Washington and Oregon?

✓ **CHECK Your Understanding**

Examples 1, 2
(pp. 518–519)

Display each set of data in a circle graph.

1.

Blood Types in the U.S.	
Blood Type	Percent
O	44%
A	42%
B	10%
AB	4%

Source: Stanford School of Medicine

2.

Favorite Musical Instrument	
Type	Number of Students
Piano	54
Guitar	27
Drum	15
Flute	24

Examples 3, 4
(p. 520)

COLORS For Exercises 3 and 4, use the graph that shows the results of a survey.

3. What color is most favored?

4. If 400 people were surveyed, how many more people favored purple than red?

Favorite Color
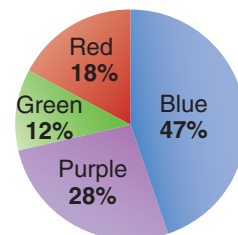
Red 18%
Green 12%
Blue 47%
Purple 28%

Practice and Problem Solving

Display each set of data in a circle graph.

5.

U.S. Steel Roller Coasters	
Type	Percent
Sit down	86%
Inverted	8%
Other	6%

6.

U.S. Orange Production	
State	Orange Production
California	18%
Florida	81%
Texas	1%

Source: National Agriculture Statistics Service

7.

Animals in Pet Store	
Animal	Number of Pets
Birds	13
Cats	11
Dogs	9
Fish	56
Other	22

8.

Favorite Games	
Type of Game	Number of Students
Card	7
Board	9
Video	39
Sports	17
Drama	8

LANDFILLS For Exercises 9-11, use the circle graph that shows what is in United States landfills.

What is in U.S. Landfills?

Metal 8%
Paper 30%
Plastic 24%
Other Trash 21%
Food and Yard Waste 11%
Rubber and Leather 6%

Source: *The World Almanac for Kids*

9. What takes up the most space in landfills?

10. About how many times more paper is there than food and yard waste?

11. If a landfill contains 200 million tons of trash, how much of it is plastic?

MONEY For Exercises 12–14, use the graph that shows the results of a survey about a common currency for North America.

Do Americans Favor Common North American Currency?

No 53%
Yes 43%
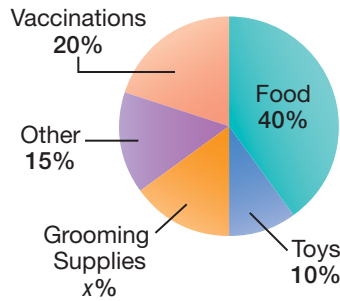Don't Know 4%

Source: Coinstar

12. What percent of Americans favor a common North American currency?

13. Based on these results, about how many of the approximately 298 million Americans would say "Don't Know" in response to this survey?

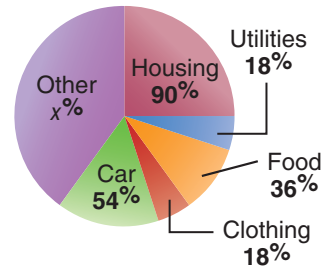14. About how many more Americans oppose a common currency than favor it?

DATA SENSE For each graph, find the missing values.

15. **Dog Expenses**

- Vaccinations 20%
- Food 40%
- Other 15%
- Grooming Supplies x%
- Toys 10%

16. **Family Budget**

- Utilities 18%
- Housing 90%
- Other x%
- Car 54%
- Food 36%
- Clothing 18%

Select an appropriate type of graph to display each set of data: line graph, bar graph, or circle graph. Then display the data using the graph.

17.

Top 5 Presidential Birth States	
Place	**Presidents**
Virginia	8
Ohio	7
Massachusetts	4
New York	4
Texas	3

Source: *The World Book of Facts*

18.

Tanya's Day	
Activity	**Percent**
School	25%
Sleep	33%
Homework	12%
Sports	8%
Other	22%

**GEOGRAPHY** For Exercises 19–21, use the table.

19. Display the data in a circle graph.

20. Use your graph to find which two lakes equal the size of Lake Superior.

21. Compare the size of Lake Ontario to the size of Lake Michigan.

Sizes of U.S. Great Lakes	
Lake	**Size (sq mi)**
Erie	9,930
Huron	23,010
Michigan	22,400
Ontario	7,520
Superior	31,820

POLITICS For Exercises 22 and 23, use the graph and information below.

A group of students were asked whether people their age could make a difference in the political decisions of elected officials.

22. How many students participated in the survey?

23. Write a convincing argument explaining whether or not it is reasonable to say that 50% more students said they could make a difference than those who said they could not make a difference.

Political Decisions

- Not Sure 2% (10 students)
- No 25% (126 students)
- Yes 73% (366 students)
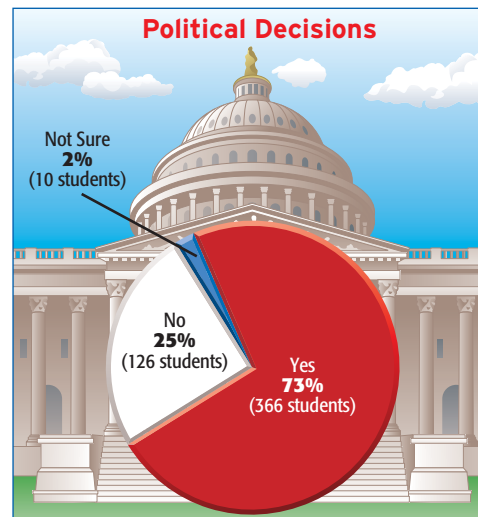
Source: *Mom's Life* and *Mothering Magazine*

24. **FIND THE DATA** Refer to the Data File on pages 16–19. Choose some data that can be displayed in a circle graph. Then display the data in a circle graph and write one statement analyzing the data.

H.O.T. Problems

25. CHALLENGE The graph shows the results of a survey about students' favorite school subject. About what percent of those surveyed said that math was their favorite subject? Explain your reasoning.

Favorite Subject

26. COLLECT THE DATA Collect some data from your classmates that can be represented in a circle graph. Then create the circle graph and write one statement analyzing the data.

27. WRITING IN MATH The table shows the percent of people that like each type of fruit juice. Can the data be represented in a circle graph? Justify your answer.
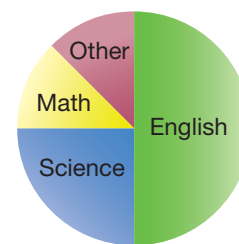
Fruit Juice	Percent
Apple	54%
Grape	48%
Orange	37%
Cranberry	15%

TEST PRACTICE

28. The graph shows the type of vehicles that used Highway 82 during one month.

Types of Vehicles
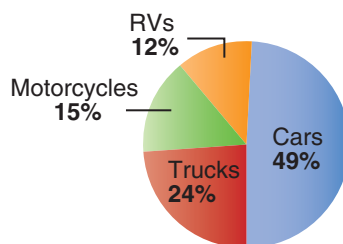
Which statement is true according to the circle graph shown?

A More cars used the highway than RVs and trucks combined.

B More than half the vehicles that used the highway were cars.

C More RVs used the highway than trucks.

D More trucks used the highway than cars.

Spiral Review

29. GEOMETRY Refer to the diagram at the right. Identify a pair of vertical angles. (Lesson 10-1)

30. ALGEBRA $\angle A$ and $\angle B$ are complementary. If $m\angle A = 15°$, find $m\angle B$. (Lesson 10-2)
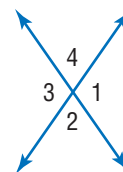
GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Lesson 3-2)

31. $x + 112 = 180$ **32.** $50 + t = 180$ **33.** $180 = 79 + y$ **34.** $180 = h + 125$

10-4 Triangles

▶ MINI Lab

STEP 1 Use a straightedge to draw a triangle with three acute angles. Label the angles *A*, *B*, and *C*. Cut out the triangle.
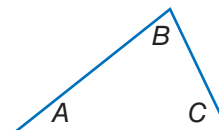
STEP 2 Fold ∠*A*, ∠*B*, and ∠*C* so the vertices meet on the line between angles *A* and *C*.
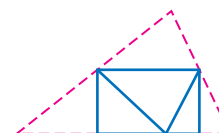
1. What kind of angle is formed where the three vertices meet?

2. Repeat the activity with another triangle. Make a conjecture about the sum of the measures of the angles of any triangle.
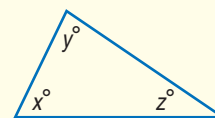
A **triangle** is a figure with three sides and three angles. The symbol for triangle is △. There is a relationship among the three angles in a triangle.

Angles of a Triangle   Key Concept

Words   The sum of the measures of the angles of a triangle is 180°.

Model

Algebra $x + y + z = 180$

EXAMPLE   Find a Missing Measure

1 **ALGEBRA** Find $m\angle Z$ in the triangle.

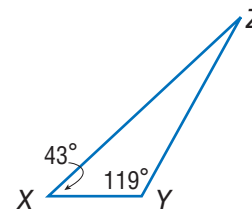Since the sum of the angle measures in a triangle is 180°, $m\angle Z + 43° + 119° = 180°$.

$$m\angle Z + 43° + 119° = 180° \quad \text{Write the equation.}$$

$$m\angle Z + 162° = 180° \quad \text{Simplify.}$$

$$\underline{-162° = -162°} \quad \text{Subtract 162° from each side.}$$

$$m\angle Z = 18°$$

So, $m\angle Z$ is 18°.

✓ CHECK Your Progress

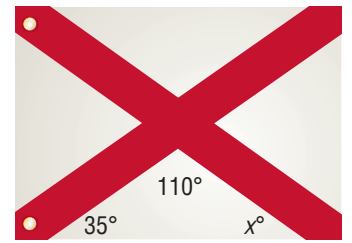a. **ALGEBRA** In △*ABC*, if $m\angle A = 25°$ and $m\angle B = 108°$, what is $m\angle C$?

TEST EXAMPLE

2 The Alabama state flag is constructed with the triangles shown. What is the missing measure in the design?

A 145°   **C** 35°

B 75°   **D** 25°

110°

35°   x°

Read the Item

To find the missing measure, write and solve an equation.

Solve the Item

$$x + 110 + 35 = 180 \quad \text{The sum of the measures is 180.}$$
$$x + 145 = 180 \quad \text{Simplify.}$$
$$\underline{-145 = -145} \quad \text{Subtract 145 from each side.}$$
$$x = 75$$

🔊 The answer is C.

✅ CHECK Your Progress

b. The frame of a bicycle shows a triangle. What is the missing measure?

F 31°   **H** 45°

G 40°   **J** 50°

89°   x°
60°
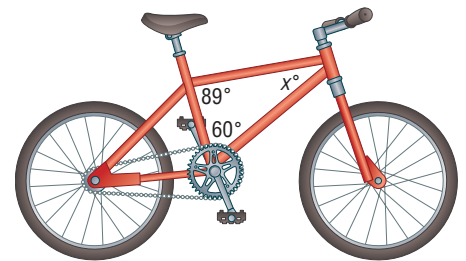
Every triangle has at least two acute angles. One way you can classify a triangle is by using the third angle. Another way to classify triangles is by their sides. Sides with the same length are **congruent segments**.
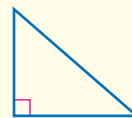
Classify Triangles Using Angles   Key Concept
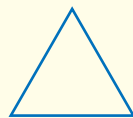
all acute angles   1 right angle   1 obtuse angle

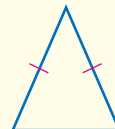**acute triangle**   **right triangle** **obtuse triangle**

Classify Triangles Using Sides

no congruent sides   at least 2 congruent sides   3 congruent sides
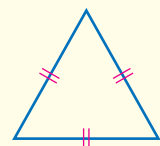
scalene triangle **isosceles triangle**   **equilateral triangle**

Study Tip

Congruent Segments
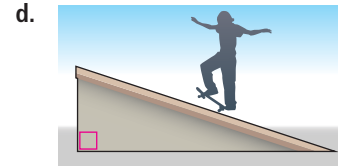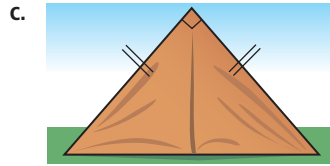The marks on the sides of the triangle indicate that those sides are congruent.

3 **Classify the marked triangle at the right by its angles and by its sides.**

The triangle on the side of a house has one obtuse angle and two congruent sides. So, it is an obtuse, isosceles triangle.
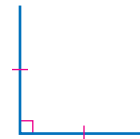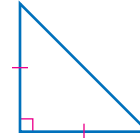
CHECK Your Progress

c.

d.

EXAMPLES **Draw Triangles**

4 **Draw a triangle with one right angle and two congruent sides. Then classify the triangle.**

Draw a right angle. The two segments should be congruent.

Connect the two segments to form a triangle.

The triangle is a right isosceles triangle.

5 **Draw a triangle with one obtuse angle and no congruent sides. Then classify the triangle.**

Draw an obtuse angle. The two segments of the angle should have different lengths.

Connect the two segments to form a triangle.

The triangle is an obtuse scalene triangle.

CHECK Your Progress

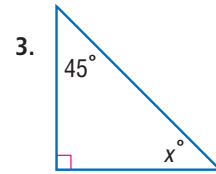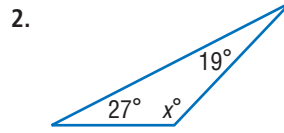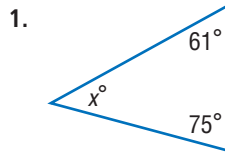**Draw a triangle that satisfies each set of conditions below. Then classify each triangle.**

e. a triangle with three acute angles and three congruent sides

f. a triangle with one right angle and no congruent sides

CHECK **Your Understanding**
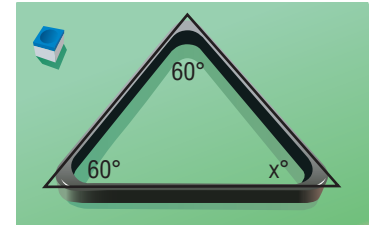
Example 1
(p. 524)

Find the value of x.

1. 61°
$x°$
75°

2. 19°
27° $x°$

3. 45°
$x°$

4. ALGEBRA Find $m\angle T$ in $\triangle RST$ if $m\angle R = 37°$ and $m\angle S = 55°$.

Example 2
(p. 525)

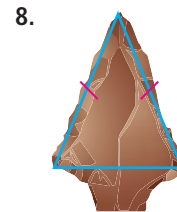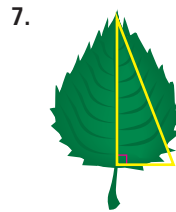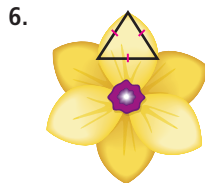**5. MULTIPLE CHOICE** A triangle is used in the game of pool to rack the pool balls. Find the missing measure of the triangle.

A 30° **C** 60°

B 40° **D** 75°

60°
60° $x°$

Example 3
(p. 526)

NATURE Classify the marked triangle in each object by its angles and by its sides.

6.

7.

8.

Examples 4, 5
(p. 526)

DRAWING TRIANGLES For Exercises 9 and 10, draw a triangle that satisfies each set of conditions. Then classify each triangle.
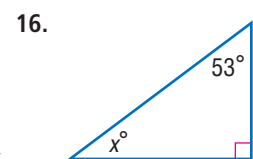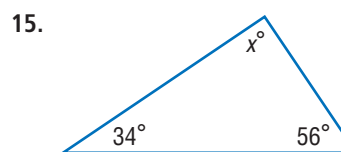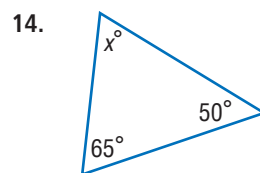
9. a triangle with three acute angles and two congruent sides

10. a triangle with one obtuse angle and two congruent sides

Practice and **Problem Solving**

HOMEWORK HELP

For Exercises	See Examples
11–18, 47, 48	1–2
19–26	3
27–30	4, 5

Find the value of x.

11. 33° $x°$
29°

12. $x°$
30°

13. 21°
132° $x°$

14. $x°$
50°
65°

15. $x°$
34° 56°

16. 53°
$x°$
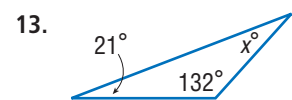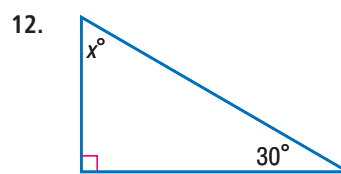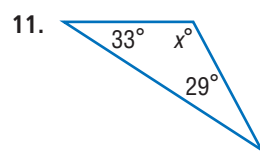
17. ALGEBRA Find $m\angle Q$ in $\triangle QRS$ if $m\angle R = 25°$ and $m\angle S = 102°$.

18. ALGEBRA In $\triangle EFG$, $m\angle F = 46°$ and $m\angle G = 34°$. What is $m\angle E$?

Classify the marked triangle in each object by its angles and by its sides.

19.

20.

21.

22.

23.

24.

25. **ART** The sculpture at the right is entitled *Texas Triangles*. It is located in Lincoln, Massachusetts. What type of triangle is shown: *acute*, *right*, or *obtuse*?

Source: DeCordova Museum and Sculpture Park

26. **ARCHITECTURE** Use the photo at the left to classify the side view of the Transamerica building by its angles and by its sides.

DRAWING TRIANGLES For Exercises 27–30, draw a triangle that satisfies each set of conditions. Then classify each triangle.
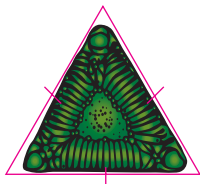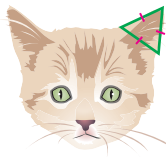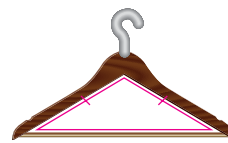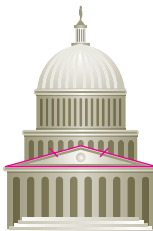
27. a triangle with three acute angles and no congruent sides

28. a triangle with one obtuse angle and two congruent sides

29. a triangle with three acute angles and three congruent sides

30. a triangle with one right angle and no congruent sides

Find the missing measure in each triangle with the given angle measures.

31. $80°$, $20.5°$, $x°$
32. $75°$, $x°$, $50.2°$
33. $x°$, $10.8°$, $90°$
34. $45.5°$, $x°$, $105.6°$
35. $x°$, $140.1°$, $18.6°$
36. $110.2°$, $x°$, $35.6°$

37. **ALGEBRA** Find the third angle measure of a right triangle if one of the angles measures is $10°$.

38. **ALGEBRA** What is the third angle measure of a right triangle if one of the angle measures is $45.8°$?

ALGEBRA Find the value of x in each triangle.

39.

40.

41.
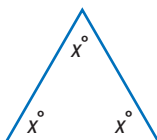
42. CHALLENGE Apply what you know about triangles to find the missing angle measures in the figure.

43. OPEN ENDED Draw an acute scalene triangle. Describe the angles and sides of the triangle.

REASONING Determine whether each statement is *sometimes*, *always*, or *never* true. Justify your answer.

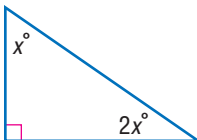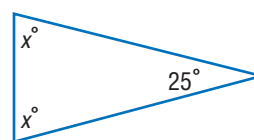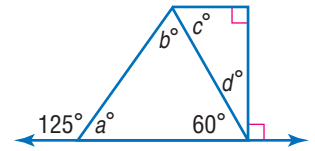**44.** It is possible for a triangle to have two right angles.

45. It is possible for a triangle to have two obtuse angles.

46. WRITING IN MATH An equilateral triangle not only has three congruent sides, but also has three congruent angles. Based on this, explain why it is impossible to draw an equilateral triangle that is either right or obtuse.

TEST PRACTICE

47. How would you find $m\angle R$?
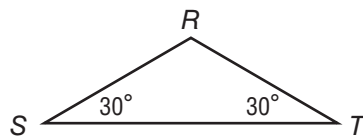
A Add 30° to 180°.

B Subtract 60° from 180°.

C Subtract 30° from 90°.

D Subtract 180° from 60°.

48. Which of the following is an acute triangle?

F

H

G

J
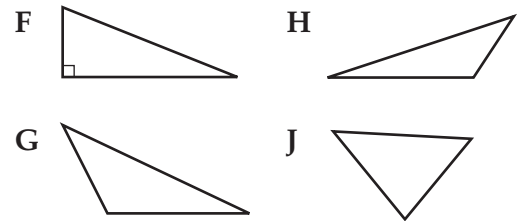
Spiral Review

49. SURVEYS A circle graph shows that 28% of people chose tea as their favorite drink. What is the measure of the angle of the tea section of the graph? (Lesson 10-3)

Classify each pair of angles as *complementary*, *supplementary*, or *neither*. (Lesson 10-2)

50.

51.

52.
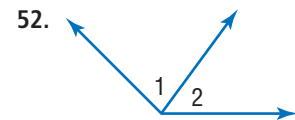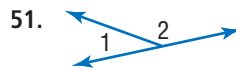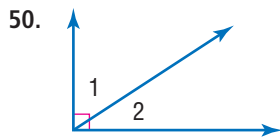
GET READY for the Next Lesson

53. PREREQUISITE SKILL If Jade buys 5 notebooks at $1.75 each, what will be the total cost of the notebooks; about $6, $7, or $9? (Lesson 7-5)

MAIN IDEA: Solve problems by using logical reasoning.
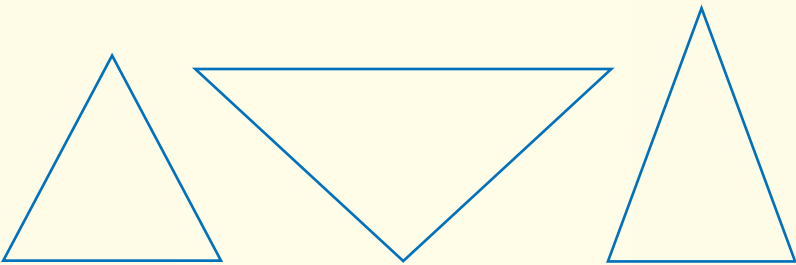
P.S.I. TEAM +

e-Mail: **USE LOGICAL REASONING**

SANTOS: I know that at least two sides of an isosceles triangle are congruent. It also looks like two of the angles in an isosceles triangle are congruent.

YOUR MISSION: Use **logical reasoning** to find if the angles in an isosceles triangles are congruent.

Understand	Isosceles triangles have at least two congruent sides. We need to find if there is a relationship between the angles.
Plan	Draw several isosceles triangles and measure their angles.
Solve	In each triangle, two angles are congruent. So, it seems like an isosceles triangle has two congruent angles.
Check	Try drawing several more isosceles triangles and measuring their angles. Although this is not a proof, it is likely your conclusion is valid.

Analyze The Strategy

1. When you use *inductive reasoning*, you make a rule after seeing several examples. When you use *deductive reasoning*, you use a rule to make a decision. What type of reasoning did Santos use to solve the problem? Explain your reasoning.

2. Explain how the *look for a pattern* strategy is similar to inductive reasoning.

Mixed Problem Solving

For Exercises 3–5, use logical reasoning to solve the problem. Justify your response.

3. **GEOMETRY** Draw several scalene triangles and measure their angles. What do you notice about the measures of the angles of a scalene triangle?

4. **HOUSE NUMBERS** Rico's house number contains four digits. The digits are 5, 8, 3, and 2. If his house number is odd, divisible by 3, and the middle two numbers are a perfect square, what is his house number?

5. **FRUIT** Julio, Rashanda, and Perry each brought a fruit with their lunch. The fruits brought were a mango, a banana, and an orange. If Perry did not bring a banana and Julio brought a mango, what type of fruit did each student bring?

**Use any strategy to solve Exercises 6–14. Some strategies are shown below.**

> PROBLEM-SOLVING STRATEGIES
> • Look for a pattern.
> • Use a graph.
> • Use logical reasoning.

6. **GEOMETRY** Draw several rectangles and measure their diagonals. Find a relationship between the diagonals of a rectangle.

7. **MONEY** Corelia has twice as many quarters in her purse as dimes and half as many nickels as dimes. If she has 12 quarters in her purse, what is the total amount of money she has?

8. **ALGEBRA** Find the next three numbers in the pattern below.

71, 64, 57, 50, ▪, ▪, ▪

9. **MEASUREMENT** The large square has been divided into 9 squares. The lengths of the squares are given. Find the area of the entire square.
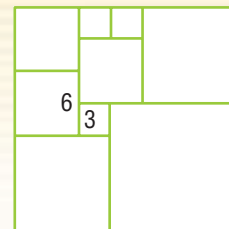
10. **READING** Sonia read 10 pages of a 150-page book on Monday. She plans to read twice as many pages each day than she did the previous day. On what day will she finish the book?

11. **SUPPLIES** Bianca has $55 to buy school supplies. She bought a backpack for $23.50, a combination lock for $6.25, and 4 binders that are $3.99 each. If the costs include tax and mechanical pencils are $2.50 per pack, how many packs can she buy?

12. **BOWLING** Colin and three of his friends are going bowling, and they have a total of $70 to spend. Suppose they buy a large pizza, four beverages, and each rent bowling shoes. How many games can they bowl if they all bowl the same number of games?

Bowling Costs	
Item	**Price**
large pizza	$15.75
beverage	$1.50
shoe rental	$3.50
game	$4.00

13. **STATISTICS** David has earned scores of 73, 85, 91, and 82 on the first four out of five math tests for the grading period. He would like to finish the grading period with a test average of at least 82. What is the minimum score David needs to earn on the fifth test in order to achieve his goal?

14. **ALLOWANCE** Samantha earns $520 in allowance yearly. Her parents promise to give her a $60 raise each year. At this rate, what will her yearly allowance be in 4 years?

Geometry Lab
Investigating Quadrilaterals

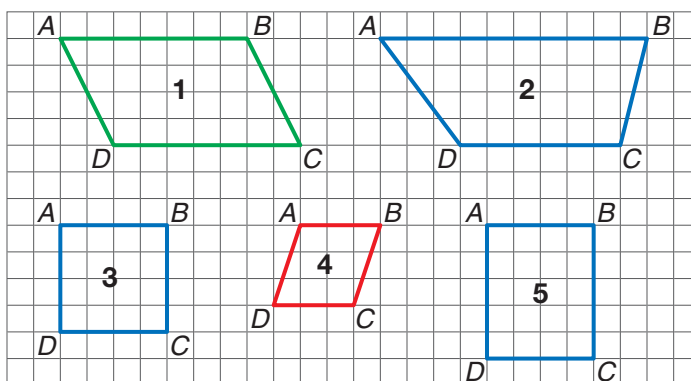**MAIN IDEA**

Investigate the properties of special quadrilaterals.

Four-sided figures are called *quadrilaterals*. In this lab, you will explore the properties of different types of quadrilaterals.

ACTIVITY

STEP 1 Draw the quadrilaterals shown on grid paper.

STEP 2 Use a ruler and a protractor to measure the sides and angles of each quadrilateral. Record your results in a table.

Quadrilateral	m∠A	m∠B	m∠C	m∠D	AB	BC	CD	DA
1								
2								

ANALYZE THE RESULTS

1. Describe any similarities or patterns in the angle measurements.

2. Describe any similarities or patterns in the side measurements.

3. **MAKE A VENN DIAGRAM** Cut out the quadrilaterals you drew in the activity. Then sort them into categories according to their similarities and differences. Arrange and record your categories in a two-circled Venn diagram. Be sure to label each circle with its category.

4. Create two other Venn diagrams illustrating two different ways of categorizing these quadrilaterals.

5. **WRITING IN MATH** Did you find shapes that did not fit a category? Where did you place these shapes? Did any shapes have properties allowing them to belong to more than one category? Could you arrange these quadrilaterals into a three-circled Venn diagram? If so, how?

Quadrilaterals

MAIN IDEA

Identify and classify quadrilaterals.

New Vocabulary

quadrilateral
parallelogram
trapezoid
rhombus

Math Online

glencoe.com

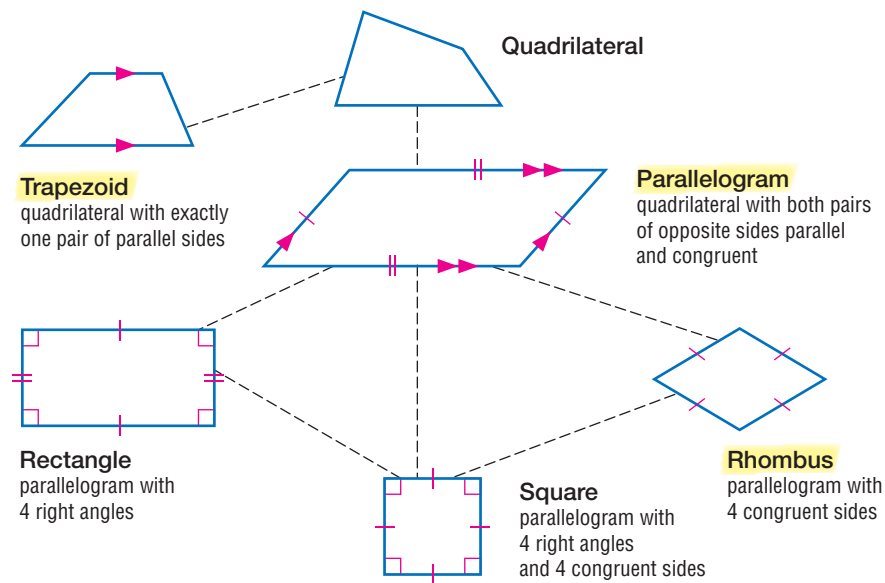• Extra Examples
• Personal Tutor
• Self-Check Quiz

▷ GET READY for the Lesson

VIDEO GAMES The general shape of a video game controller is shown.

1. Describe the angles inside the four-sided figure.

2. Which sides of the figure appear to be parallel?

3. Which sides appear to be congruent?

A **quadrilateral** is a closed figure with four sides and four angles. Quadrilaterals are named based on their sides and angles. The diagram shows how quadrilaterals are related. Notice how it goes from the most general to the most specific.

Quadrilateral

Trapezoid
quadrilateral with exactly one pair of parallel sides

Parallelogram
quadrilateral with both pairs of opposite sides parallel and congruent

Rectangle
parallelogram with 4 right angles

Square
parallelogram with 4 right angles and 4 congruent sides

Rhombus
parallelogram with 4 congruent sides

The name that *best* describes a quadrilateral is the one that is most specific.

• If a quadrilateral has all the properties of a parallelogram and a rhombus, then the *best* description of the quadrilateral is a rhombus.

• If a quadrilateral has all the properties of a parallelogram, rhombus, rectangle, and square, then the *best* description of the quadrilateral is a square.

Study Tip

Parallel Lines The sides with matching arrows are parallel.
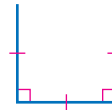
EXAMPLES **Draw and Classify Quadrilaterals**

Draw a quadrilateral that satisfies each set of conditions. Then classify each quadrilateral with the name that best describes it.

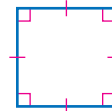**1** **a parallellogram with four right angles and four congruent sides**

Draw one right angle. The two segments should be congruent.

Draw a second right angle that shares one of the congruent segments. The third segment drawn should be congruent to the first two segments drawn.

Connect the fourth side of the quadrilateral. All four angles should be right angles, and all four sides should be congruent.

The figure is a square.

Study Tip

Check for Reasonableness
Use a ruler and a protractor to measure the sides and angles to verify that your drawing satisfies the given conditions.

2 **a quadrilateral with opposite sides parallel**

Draw two parallel sides of equal length.

Connect the endpoints of these two sides so that two new parallel sides are drawn.
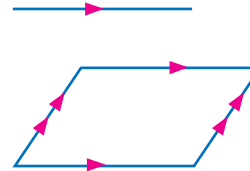
The figure is a parallelogram.

✓ CHECK Your Progress

Draw a quadrilateral that satisfies each set of conditions. Then classify each quadrilateral with the name that best describes it.

a. a quadrilateral with exactly one pair of parallel sides

b. a parallelogram with four congruent sides

A quadrilateral can be separated into two triangles, *A* and *B*. Since the sum of the angle measures of each triangle is 180°, the sum of the angle measures of the quadrilateral is 2 · 180, or 360°.
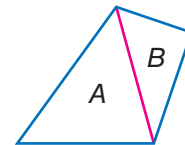
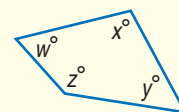## Angles of a Quadrilateral   **Key Concept**

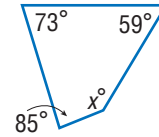| Words	The sum of the measures of the angles of a quadrilateral is 360°.	Model
Algebra	$w + x + y + z = 360$	

EXAMPLE   Find a Missing Measure

3 **ALGEBRA** Find the value of x in the quadrilateral shown.

Write and solve an equation.

Words	The sum of the measures is 360°.
Variable	Let x represent the missing measure.
Equation	$85 + 73 + 59 + x = 360$

Study Tip

Check for Reasonableness
Since $\angle x$ is an obtuse angle, $m\angle x$ should be between 90° and 180°. Since $90° < 143° < 180°$, the answer is reasonable.

$$85 + 73 + 59 + x = 360 \qquad \text{Write the equation.}$$
$$217 + x = 360 \qquad \text{Simplify.}$$
$$\underline{-217 \qquad = -217} \qquad \text{Subtract 217 from each side.}$$
$$x = 143$$

So, the missing angle measure is 143°.

CHECK Your Progress

c. **ALGEBRA** Find the value of x in the quadrilateral shown.
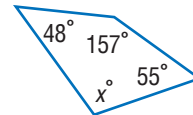
CHECK Your Understanding

Examples 1, 2
(p. 534)

Classify each quadrilateral with the name that best describes it.
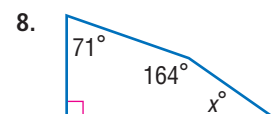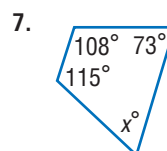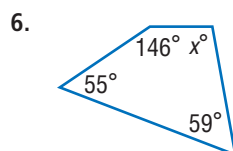
1.

2.

3.
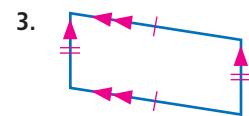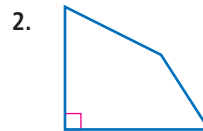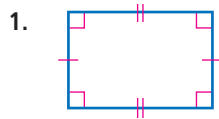
4. **BOATS** The photo shows a sailboat called a schooner. What type of quadrilateral does the indicated sail best represent?

Example 3
(p. 535)

5. **ALGEBRA** In quadrilateral $DEFG$, $m\angle D = 57°$, $m\angle E = 78°$, $m\angle G = 105°$. What is $m\angle F$?

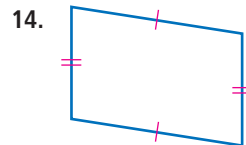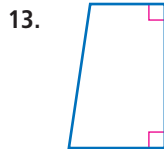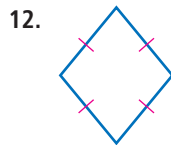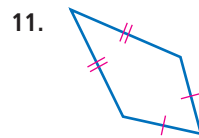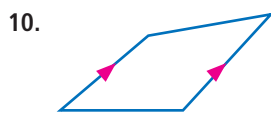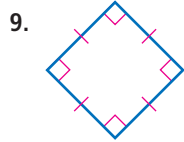**ALGEBRA** Find the missing angle measure in each quadrilateral.

6.

7.

8.

Practice and Problem Solving

Classify each quadrilateral with the name that best describes it.

9.

10.

11.

12.

13.

14.

ALGEBRA Find the missing angle measure in each quadrilateral.

15. 106°, $x°$, 131°, 67°

16. 80°, 89°, $x°$, 72°

17. $x°$, 113°

18. $x°$, 32°, 55°, 143°

19. $x°$

20. 100°, $x°$, 80°, 100°
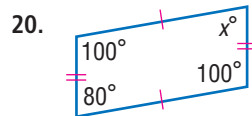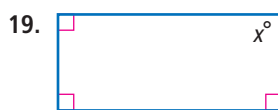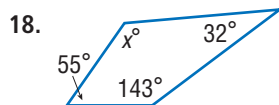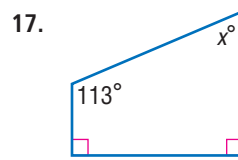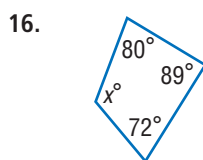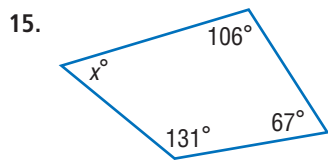
21. **ALGEBRA** Find $m\angle B$ in quadrilateral $ABCD$ if $m\angle A = 87°$, $m\angle C = 135°$, and $m\angle D = 22°$.

22. **ALGEBRA** What is $m\angle X$ in quadrilateral $WXYZ$ if $m\angle W = 45°$, $m\angle Y = 128°$, and $\angle Z$ is a right angle?

23. **LANDSCAPE** Identify the shapes of the bricks used in the design at the right. Use the name that *best* describes each brick.
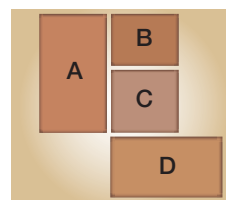
24. **MEASUREMENT** Find each of the missing angle measures a, b, c, and d in the figure at the right. Justify your answers.
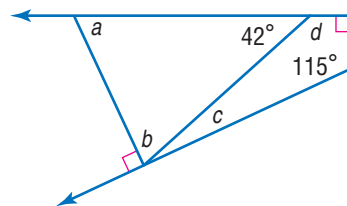
Find the missing measure in each quadrilateral with the given angle measures.

25. 37.5°, 78°, 115.4°, $x°$

26. $x°$, 108.3°, 49.8°, 100°

27. 25.5°, $x°$, 165.9°, 36.8°

28. 79.1°, 120.8°, $x°$, 65.7°

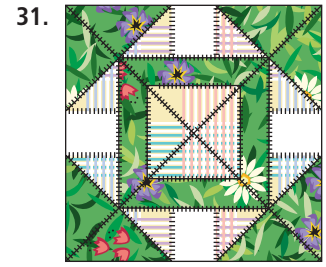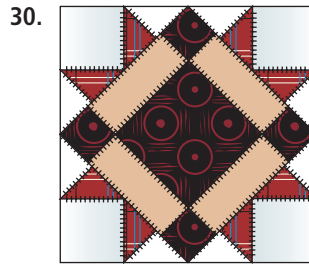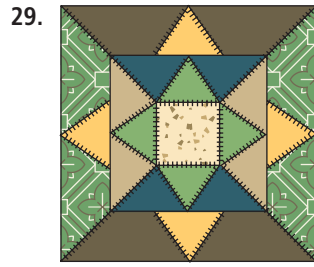**ART** For Exercises 29–31, identify the types of triangles and quadrilaterals used in each quilt block pattern. Use the names that *best* describe the figures.

29.

30.

31.

DRAWING QUADRILATERALS Determine whether each figure described below can be drawn. If the figure can be drawn, draw it. If not, explain why not.

32. a quadrilateral that is both a rhombus and a rectangle

33. a trapezoid with three right angles

34. a trapezoid with two congruent sides

ALGEBRA Find the value of x in each quadrilateral.

35.

36.

37.

H.O.T. Problems

CHALLENGE For Exercises 38 and 39, refer to the table that gives the properties of several parallelograms. Property A states that both pairs of opposite sides are parallel and congruent.

Parallelogram	Properties
1	A, C
2	A, B, C
3	A, B

38. If property C states that all four sides are congruent, classify parallelograms 1–3. Justify your response.

39. If parallelogram 3 is a rectangle, describe Property B. Justify your response.

REASONING Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

40. A quadrilateral is a trapezoid.

41. A trapezoid is a parallelogram.

42. A square is a rectangle.

43. A rhombus is a square.

44. **FIND THE ERROR** Isabelle and John are describing a rectangle. Who is more accurate? Explain.

a quadrilateral with 2 pairs of parallel sides

Isabelle

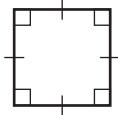*a parallelogram with 4 right angles*

John

45. **WRITING IN MATH** The diagonals of a rectangle are congruent, and the diagonals of a rhombus are perpendicular. Based on this information, what can you conclude about the diagonals of a square? of a parallelogram? Explain your reasoning.

TEST PRACTICE

46. Identify the name that does *not* describe the quadrilateral shown.

  A square

  B rectangle

  C rhombus

  D trapezoid

47. Which statement is always true about a rhombus?

  F It has 4 right angles.

  G The sum of the measures of the angles is 180°.

  H It has exactly one pair of parallel sides.

  J It has 4 congruent sides.

48. Which of the following is a correct drawing of a quadrilateral with all sides congruent and with four right angles?

  A

  B

  C

  D
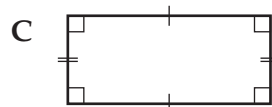
Spiral Review

49. REASONING Neva, Sophie, and Seth have a turtle, a dog, and a hamster for a pet, but not in that order. Sophie's pet lives in a glass aquarium and does not have fur. Neva never has to give her pet a bath. Who has what pet? Use the *logical reasoning* strategy. (Lesson 10-5)

Classify each triangle by its angles and by its sides. (Lesson 10-4)

50.   **51.**   **52.**
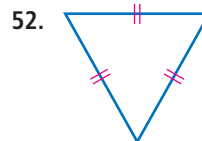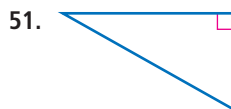
53. LETTERS How many permutations are possible of the letters in the word *Fresno?* (Lesson 9-4)

Find the sales tax or discount to the nearest cent. (Lesson 7-7)

54. $54 jacket; 7% sales tax   **55.** $23 hat; 15% discount

▶ GET READY for the Next Lesson

PREREQUISITE SKILL Solve each proportion. (Lesson 6-6)

56. $\frac{3}{5} = \frac{x}{75}$ **57.** $\frac{a}{7} = \frac{18}{42}$ **58.** $\frac{7}{9} = \frac{28}{m}$ **59.** $\frac{3.5}{t} = \frac{16}{32}$ **60.** $\frac{3}{6} = \frac{c}{5}$
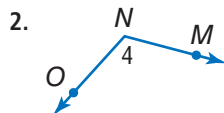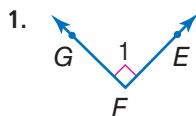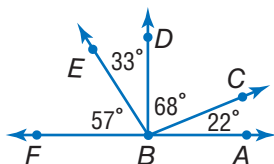
Name each angle in four ways. Then classify each angle as *acute*, *obtuse*, *right*, or *straight*.
(Lesson 10-1)

1.

2.

3. **MULTIPLE CHOICE** Which angle is complementary to $\angle CBD$? (Lesson 10-2)
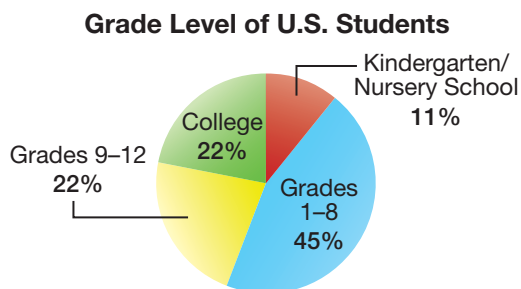
A $\angle ABC$ **C** $\angle DBE$

B $\angle FBC$ **D** $\angle EBF$

4. **SOCCER** Display the data in a circle graph. (Lesson 10-3)

Injuries of High School Girls' Soccer Players	
Position	**Percent**
Halfbacks	37%
Fullbacks	23%
Forward Line	28%
Goalkeepers	12%

EDUCATION For Exercises 5 and 6, use the circle graph that shows the percent of students by grade level in U.S. schools. (Lesson 10-3)

Grade Level of U.S. Students

5. In which grades are most students?
6. How many times as many students are there in grades 1–8 than in grades 9–12?
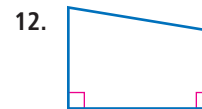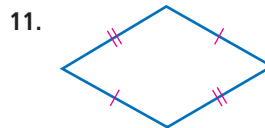
ALGEBRA Find the value of x. (Lesson 10-4)

7.

8.
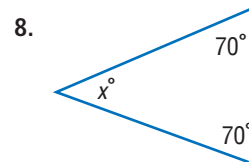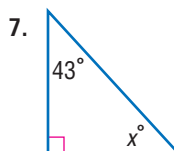
9. **MULTIPLE CHOICE** In triangle ABC, $m\angle A = 62°$ and $m\angle C = 44°$. What is $m\angle B$? (Lesson 10-4)

F 90°   **H** 64°

G 74°   **J** 42°

10. **RACES** Norberto, Isabel, Fiona, Brock, and Elizabeth were the first five finishers of a race. From the given clues, find the order in which they finished. Use the *logical reasoning* strategy. (Lesson 10-5)

- Norberto passed Fiona just before the finish line.
- Elizabeth finished 5 seconds ahead of Norberto.
- Isabel crossed the finish line after Fiona.
- Brock was fifth at the finish line.

Classify the quadrilateral with the name that best describes it. (Lesson 10-6)

11.

12.

ALGEBRA Find the value of x in each quadrilateral. (Lesson 10-6)

13.

14.
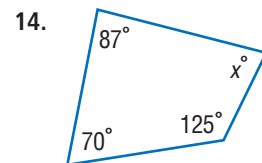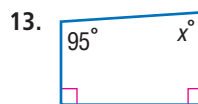
15. **ALGEBRA** What is $m\angle A$ in quadrilateral $ABCD$ if $m\angle B = 36°$, $m\angle C = 74°$, and $\angle D$ is a right angle? (Lesson 10-6)

MAIN IDEA

Determine whether figures are similar and find a missing length in a pair of similar figures.

New Vocabulary

similar figures
corresponding sides
corresponding angles
indirect measurement

Math Online

glencoe.com

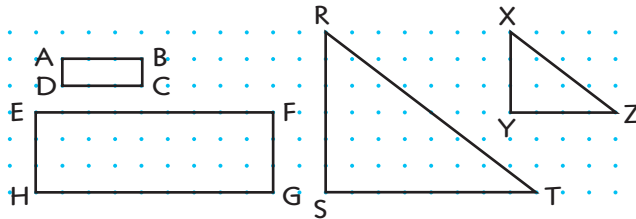• Extra Examples
• Personal Tutor
• Self-Check Quiz

▶ **MINI Lab**

The figures in each pair below have the same shape but different sizes. Copy each pair onto dot paper. Then find the measure of each angle using a protractor and the measure of each side using a centimeter ruler.

1. $\overline{AB}$ on the smaller rectangle matches $\overline{EF}$ on the larger rectangle. Name all pairs of matching sides in each pair of figures.

   The notation $\overline{AB}$ means the segment with endpoints at A and B.

2. Write each ratio in simplest form.

   The notation AB means the *measure* of segment AB.

   a. $\dfrac{AB}{EF}$, $\dfrac{BC}{FG}$, $\dfrac{DC}{HG}$, $\dfrac{AD}{EH}$ b. $\dfrac{RS}{XY}$, $\dfrac{ST}{YZ}$, $\dfrac{RT}{XZ}$

3. What do you notice about the ratios of matching sides?

4. Name all pairs of matching angles in the figures above. What do you notice about the measure of these angles?

5. **MAKE A CONJECTURE** about figures that have the same shape but not necessarily the same size.

Figures that have the same shape but not necessarily the same size are **similar figures**. In the figures below, triangle RST is similar to triangle XYZ. We write this as $\triangle RST \sim \triangle XYZ$.
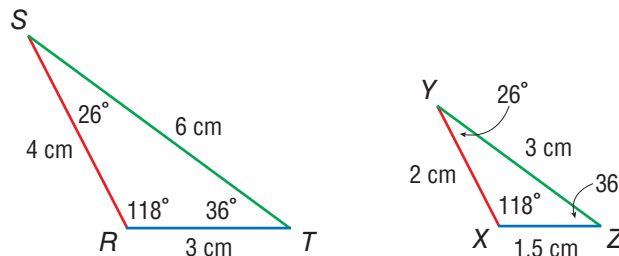
The matching sides are $\overline{ST}$ and $\overline{YZ}$, $\overline{SR}$ and $\overline{YX}$, and $\overline{RT}$ and $\overline{XZ}$. The sides of similar figures that "match" are called **corresponding sides**.

The matching angles are $\angle S$ and $\angle Y$, $\angle R$ and $\angle X$, and $\angle T$ and $\angle Z$. The angles of similar figures that "match" are called **corresponding angles**.
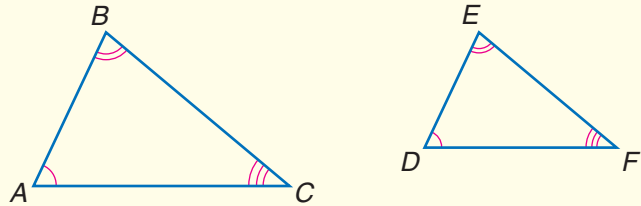
The Mini Lab illustrates the following statements.

Similar Figures

Words   If two figures are similar, then
- the corresponding sides are proportional, and
- the corresponding angles are congruent.

Models

Reading Math

Geometry Symbols

~ is similar to
≅ is congruent to

Symbols $\triangle ABC \sim \triangle DEF$ The symbol ~ means *is similar to*.

corresponding sides: $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

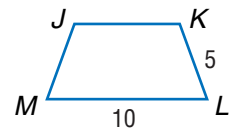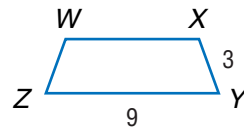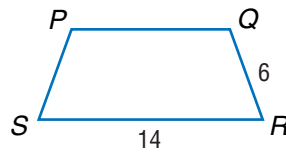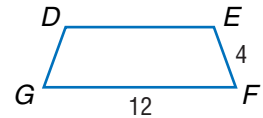corresponding angles: $\angle A \cong \angle D$; $\angle B \cong \angle E$; $\angle C \cong \angle F$

EXAMPLE   **Identify Similar Figures**

1 Which trapezoid below is similar to trapezoid *DEFG*?

Find the ratios of the corresponding sides to see if they form a constant ratio.

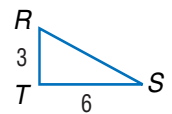| **Trapezoid *PQRS***	**Trapezoid *WXYZ***	**Trapezoid *JKLM***
$\dfrac{EF}{QR} = \dfrac{4}{6}$ or $\dfrac{2}{3}$	$\dfrac{EF}{XY} = \dfrac{4}{3}$	$\dfrac{EF}{KL} = \dfrac{4}{5}$
$\dfrac{FG}{RS} = \dfrac{12}{14}$ or $\dfrac{6}{7}$	$\dfrac{FG}{YZ} = \dfrac{12}{9}$ or $\dfrac{4}{3}$	$\dfrac{FG}{LM} = \dfrac{12}{10}$ or $\dfrac{6}{5}$
Not similar	Similar	Not similar

So, trapezoid *WXYZ* is similar to trapezoid *DEFG*.

CHECK Your Progress
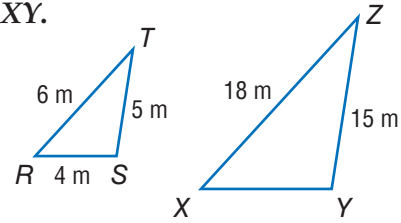
a. Which triangle below is similar to triangle *DEF*?
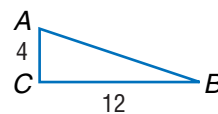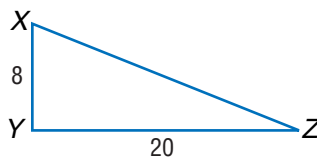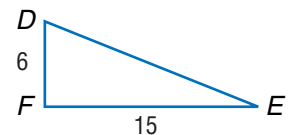
EXAMPLE **Find Side Measures of Similar Triangles**

2 **If** $\triangle RST \sim \triangle XYZ$, **find the length of** $\overline{XY}$.

Since the two triangles are similar, the ratios of their corresponding sides are equal. Write and solve a proportion to find XY.

Reading Math

Geometry Symbols

Just as the measure of angle A can be written as $m\angle A$, there is a special way to indicate the measure of a segment. The measure of $\overline{AB}$ is written as AB, without the bar over it.

$$\frac{RT}{XZ} = \frac{RS}{XY} \qquad \text{Write a proportion.}$$

$$\frac{6}{18} = \frac{4}{n} \qquad \text{Let } n \text{ represent the length of } \overline{XY}. \text{ Then substitute.}$$

$$6n = 18(4) \qquad \text{Find the cross products.}$$

$$6n = 72 \qquad \text{Simplify.}$$

$$n = 12 \qquad \text{Divide each side by 6.} \quad \text{The length of } \overline{XY} \text{ is 12 meters.}$$

✓ **CHECK Your Progress**

**b.** If $\triangle ABC \sim \triangle EFD$, find the length of $\overline{AC}$.
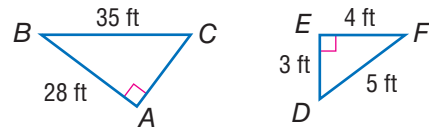
Indirect measurement uses similar figures to find the length, width, or height of objects that are too difficult to measure directly.

🌎 **Real-World EXAMPLE**

3 **GEYSERS** Old Faithful in Yellowstone National Park shoots water 60 feet into the air that casts a shadow of 42 feet. What is the height of a nearby tree that casts a shadow 63 feet long? Assume the triangles are similar.
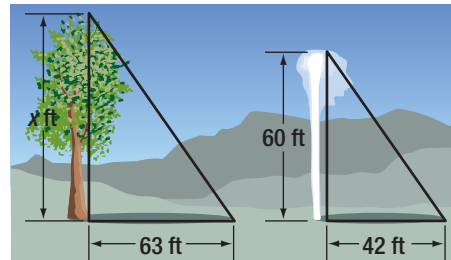
Tree   Old Faithful

$$\frac{x}{63} = \frac{60}{42} \qquad \begin{array}{l}\leftarrow \text{ height} \\ \leftarrow \text{ shadow}\end{array}$$

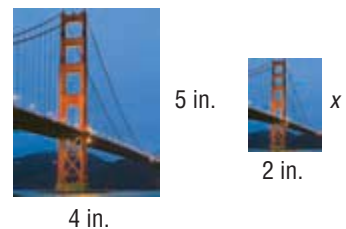$$42x = 60(63) \quad \text{Find the cross products.}$$

$$42x = 3{,}780 \quad \text{Simplify.}$$

$$x = 90 \qquad \text{Divide each side by 42.}$$

The tree is 90 feet tall.

✓ **CHECK Your Progress**

c. **PHOTOGRAPHY** Destiny wants to resize a 4-inch wide by 5-inch long photograph for the school newspaper. It is to fit in a space that is 2 inches wide. What is the length of the resized photograph?
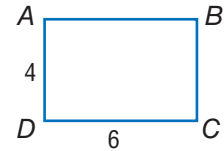
Example 1
(p. 541)

1. Which rectangle below is similar to rectangle *ABCD*?
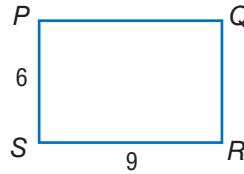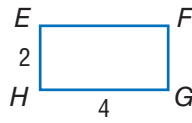
Example 2
(p. 542)

ALGEBRA Find the value of *x* in each pair of similar figures.

2.
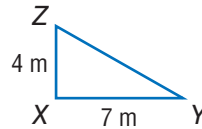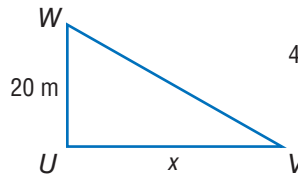
3.
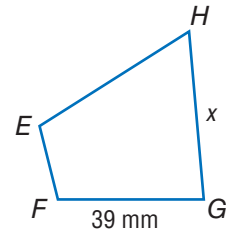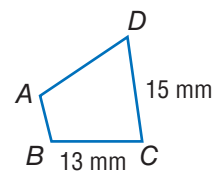
Example 3
(p. 542)

4. **SHADOWS** A flagpole casts a 20-foot shadow. At the same time, Humberto, who is 6 feet tall, casts a 5-foot shadow. What is the height of the flagpole? Assume the triangles are similar.
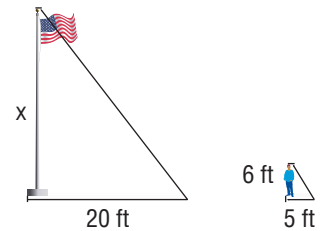
Practice and Problem Solving

For Exercises	See Examples
5–6	1
7–10	2
11–12	3

HOMEWORK HELP

5. Which triangle below is similar to triangle *FGH*?
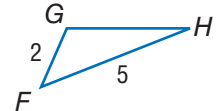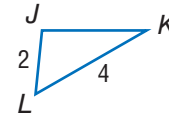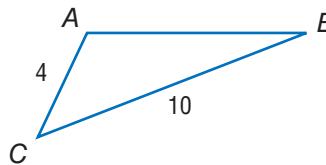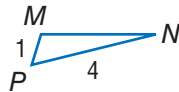
6. Which parallelogram below is similar to parallelogram *HJKM*?
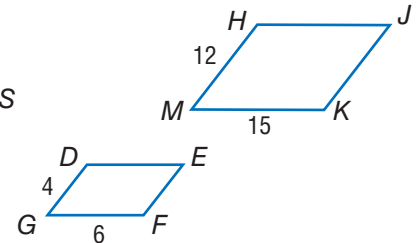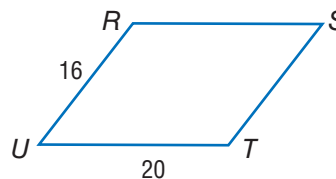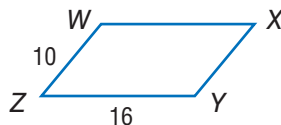
ALGEBRA Find the value of *x* in each pair of similar figures.

7.

8.

ALGEBRA Find the value of x in each pair of similar figures.

9.

x
12 in.
5 in.
3 in.

10.

4 m
18 m
9 m
x
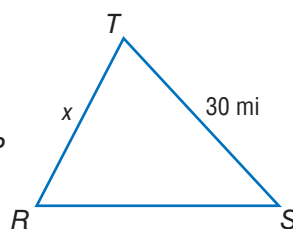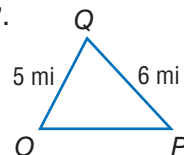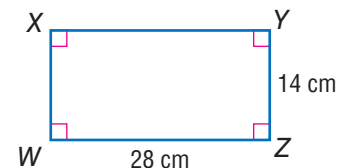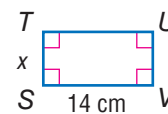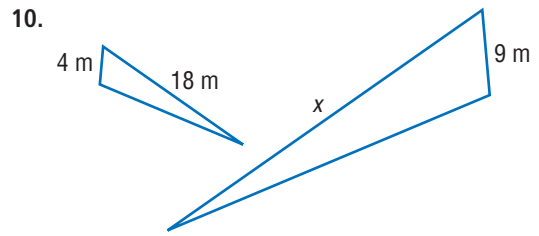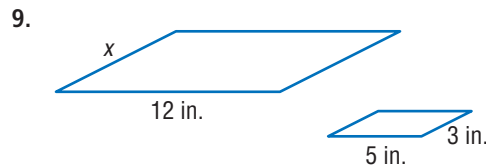
11. **PARKS** Ruth is at the park standing next to a slide. Ruth is 5 feet tall, and her shadow is 4 feet long. If the shadow of the slide is 4.8 feet long, what is the height of the slide? Assume the triangles are similar.

5 ft
x ft
◀— 4 ft —▶
◀— 4.8 ft —▶
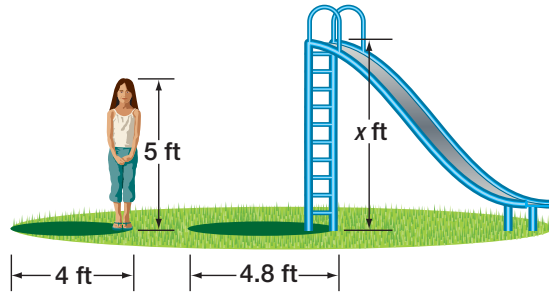
12. **FURNITURE** A child's desk is made so that it is a replica of a full-size adult desk. Suppose the top of the full-size desk measures 54 inches long by 36 inches wide. If the top of a child's desk is 24 inches wide and is similar to the full-size desk, what is the length?


◀— x in. —▶
24 in.
◀— 54 in. —▶
36 in.

ALGEBRA Find the value of x in each pair of similar figures.

13.

A
29.4 m
B
25.2 m
C
D
14 m
E
x
F

14.

F
E
7.2 mm
H
G
x
K
5.1 mm
L
M
J
3.6 mm
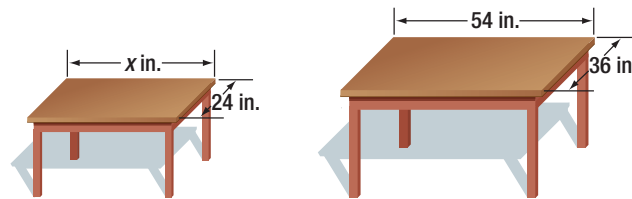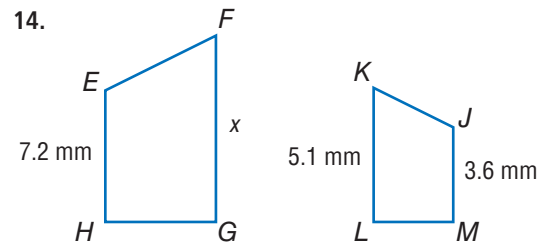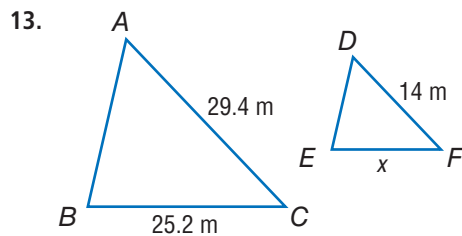
SKYCRAPERS For Exercises 15 and 16, use the information below and at the left.

Tricia has a miniature replica of the Empire State Building. The replica is 10 inches tall, and the height of the observation deck is 8.3 inches.

15. About how tall is the actual observation deck?

16. Tricia's sister has a larger replica in which the height of the observation deck is 12 inches. How tall is the larger replica?

17. **MEASUREMENT** The ratio of the length of square A to the length of square B is 3:5. If the length of square A is 18 meters, what is the perimeter of square B?

H.O.T. Problems   **CHALLENGE** For Exercises 18 and 19, use the following information.

Two rectangles are similar. The ratio of their corresponding sides is 1:4.

18. Find the ratio of their perimeters.

19. What is the ratio of their areas?

20. **WRITING IN** **MATH** Write a problem about a real-world situation that could be solved using proportions and the concept of similarity. Then use what you have learned in this lesson to solve the problem.

TEST PRACTICE

21. Which rectangle is similar to the rectangle shown?

2 in.

1 in.

A   3 ft

3 ft

B   16 yd

8 yd

C   28 yd

8 yd
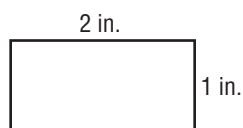
D   12 m

4 m

22. Which of the following equations is a correct use of cross-multiplication in solving the proportion $\frac{12}{15} = \frac{m}{6}$?

F $12 \cdot m = 15 \cdot 6$

G $12 \cdot 6 = m \cdot 15$

H $12 \cdot 15 = m \cdot 6$

J $12 \div 6 = m \div 15$

23. Horatio is 6 feet tall and casts a shadow 3 feet long. What is the height of a nearby tower if it casts a shadow 25 feet long at the same time?

h

|◄25 ft►|

6 ft

►|3 ft|◄
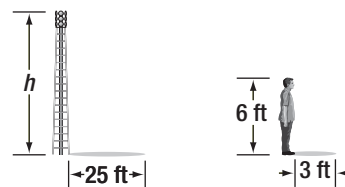
A 25 feet   **C** 50 feet

B 45 feet   **D** 75 feet

Spiral Review

GEOMETRY Classify the quadrilateral using the name that *best* describes it. (Lesson 10-6)

24.

25.

26.
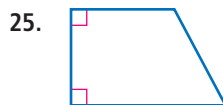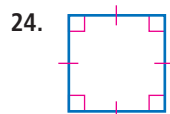
27. **MEASUREMENT** A triangular-shaped sail has angle measures of 44° and 67°. Find the measure of the third angle. (Lesson 10-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Lesson 3-3)

28. $5a = 120$ **29.** $360 = 4x$ **30.** $940 = 8n$ **31.** $6t = 720$

Polygons and Tessellations

MAIN IDEA

Classify polygons and determine which polygons can form a tessellation.

New Vocabulary

polygon
pentagon
hexagon
heptagon
octagon
nonagon
decagon
regular polygon
tessellation

▶ **GET READY** for the Lesson

POOLS Prairie Pools designs and builds swimming pools in various shapes and sizes. The shapes of five swimming pool styles are shown in their catalog.

Aquarius   Kidney   Roman   Oval   Rustic

1. In the pool catalog, the Aquarius and the Roman styles are listed under Group A. The remaining three pools are listed under Group B. Describe one difference between the shapes of the pools in the two groups.

2. Create your own drawing of the shape of a pool that would fit into Group A. Group B.

A **polygon** is a simple, closed figure formed by three or more straight line segments. A *simple figure* does not have lines that cross each other. You have drawn a *closed figure* when your pencil ends up where it started.

Polygons	Not Polygons
• Line segments are called sides. • Sides meet only at their endpoints. • Points of intersection are called vertices.	• Figures with sides that cross each other. • Figures that are open. • Figures that have curved sides.

A polygon can be classified by the number of sides it has.

Words	pentagon	hexagon	heptagon	octagon	nonagon	decagon
Number of Sides	5	6	7	8	9	10
Models						

A **regular polygon** has all sides congruent and all angles congruent. Equilateral triangles and squares are examples of regular polygons.

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is not a polygon, explain why.
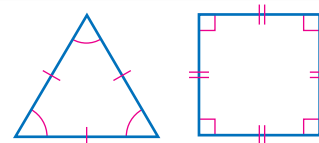
Reading Math

Regular Polygons Since regular polygons have *equal–sized angles*, they are also called *equiangular*.

1
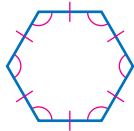
The figure has 6 congruent sides and 6 congruent angles. It is a regular hexagon.

2
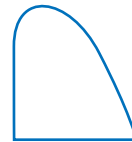
The figure is not a polygon since it has a curved side.

CHECK Your Progress

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

a.

b.
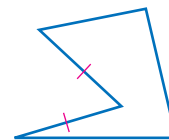
The sum of the measures of the angles of a triangle is 180°. You can use this relationship to find the measures of the angles of regular polygons.

EXAMPLE **Angle Measures of a Polygon**

Study Tip

Angle Measures The number of triangles formed is 2 less than the number of sides in the polygon. The equation $(n - 2) + 180 = s$ gives the sum s of angle measures in a polygon with n sides.

3 **ALGEBRA** Find the measure of each angle of a regular pentagon.

• Draw all of the diagonals from one vertex as shown and count the number of triangles formed.

• Find the sum of the angle measures in the polygon.

number of triangles formed × 180° = sum of angle measures in polygon

$$3 \times 180° = 540°$$

• Find the measure of each angle of the polygon. Let n represent the measure of one angle in the pentagon.

$5n = 540$ There are five congruent angles.

$n = 108$ Divide each side by 5.

The measure of each angle in a regular pentagon is 108°.

CHECK Your Progress

Find the measure of an angle in each polygon.

c. regular octagon   **d.** equilateral triangle

A repetitive pattern of polygons that fit together with no overlaps or holes is called a **tessellation**. The surface of these bricks is an example of a tessellation of squares.

The sum of the measures of the angles where the vertices meet in a tessellation is 360°.

$$90° \quad 90°$$
$$90° \quad 90°$$
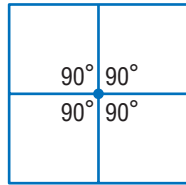
$$4 \times 90° = 360°$$

Real-World EXAMPLE

4 **DESIGN** Ms. Evans wants to design a floor tile using pentagons. Can Ms. Evans make a tessellation using pentagons?

The measure of each angle in a regular pentagon is 108°.

The sum of the measures of the angles where the vertices meet must be 360°. So, solve $108°n = 360$.

$108n = 360$ Write the equation.

$\dfrac{108n}{108} = \dfrac{360}{108}$ Divide each side by 108.

$n = 3.3$ Use a calculator.

108°

Since 108° does not divide evenly into 360°, the sum of the measures of the angles where the vertices meets is not 360°. So, Ms. Evans cannot make a tessellation using the pentagons.

Check

108° 108°
108°

Math Online
For more information, go to glencoe.com.
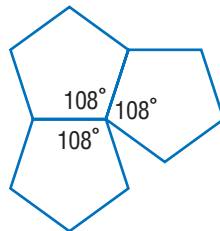
CHECK Your Progress

e. **DESIGN** Can Ms. Evans use tiles that are equilateral triangles to cover the floor? Explain.

Examples 1, 2
(p. 547)

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

1.

2.

3.
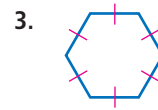
Example 3
(p. 547)

Find the measure of an angle in each polygon if the polygon is regular. Round to the nearest tenth of a degree if necessary.

4. hexagon

5. heptagon

Example 4
(p. 548)

6. **ART** In art class, Trisha traced and then cut several regular octagons out of tissue paper. Can she use the figures to create a tessellation? Explain.
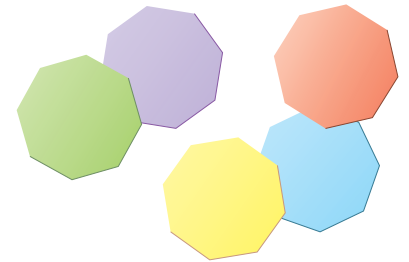
Practice and **Problem Solving**

For Exercises	See Examples
7–12	1, 2
13–16	3
17–18	4

HOMEWORK HELP

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

7.

8.

9.

10.

11.

12.
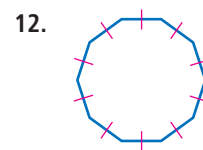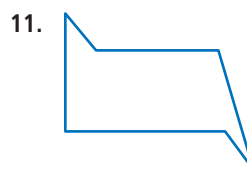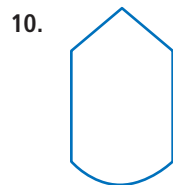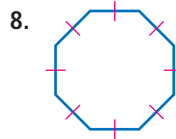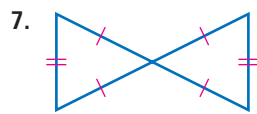
Find the measure of an angle in each polygon if the polygon is regular. Round to the nearest tenth of a degree if necessary.

13. decagon

14. nonagon

15. quadrilateral

16. 11-gon

17. **TOYS** Marty used his magnetic building set to build the regular decagon at the right. Assume he has enough building parts to create several of these shapes. Can the figures be arranged in a tessellation? Explain.
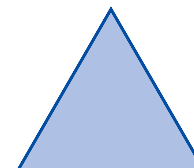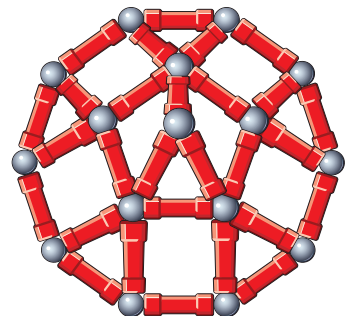
18. **COASTERS** Paper coasters are placed under a beverage glass to protect the table surface. The coasters are shaped like regular heptagons. Can the coasters be arranged in a tessellation? Explain your reasoning.

Classify the polygons that are used to create each tessellation.

19.
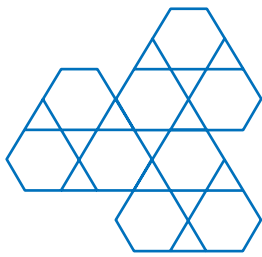
20.
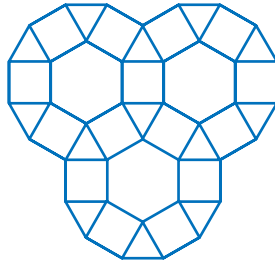
21.
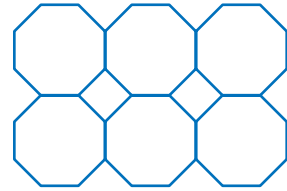
22. What is the perimeter of a regular nonagon with sides 4.8 centimeters?

23. Find the perimeter of a regular pentagon having sides $7\frac{1}{4}$ yards long.

24. **ART** The mosaic shown at the right is from a marble floor in Venice, Italy. Name the polygons used in the floor.

25. **SIGNS** Refer to the photo at the left. Stop signs are made from large sheets of steel. Suppose one sheet of steel is large enough to cut nine signs. Can all nine signs be arranged on the sheet so that none of the steel goes to waste? Explain.

26. **RESEARCH** Use the Internet or another source to find the shape of other road signs. Name the type of sign, its shape, and state whether or not it is regular.

SCHOOL For Exercises 27–29, use the information below and the graphic of the cafeteria tray shown.

A company designs cafeteria trays so that four students can place their trays around a square table without bumping corners. The top and bottom sides of the tray are parallel.
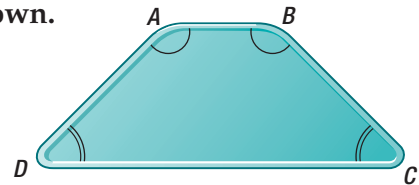
27. Classify the shape of the tray.

28. If $\angle A \cong \angle B$, $\angle C \cong \angle D$, and $m\angle A = 135°$, find $m\angle B$, $m\angle C$, and $m\angle D$.

29. Name the polygon formed by the outside of four trays when they are placed around the table with their sides touching. Justify your answer.

H.O.T. Problems

30. **REASONING** *True* or *False*? Only a regular polygon can tessellate a plane.

31. **OPEN ENDED** Draw examples of a pentagon and a hexagon that represent real-world objects.

32. **CHALLENGE** You can make a tessellation with equilateral triangles. Can you make a tessellation with any isosceles or scalene triangles? If so, explain your reasoning and make a drawing.

33. **WRITING IN MATH** Analyze the parallelogram at the right and then explain how you know the parallelogram can be used by itself to make a tessellation.
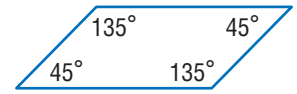
TEST PRACTICE

34. **SHORT RESPONSE** What is the measure of $\angle 1$?
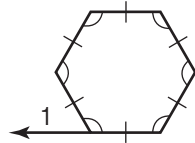
35. Which statement is *not* true about polygons?

    A A polygon is classified by the number of sides it has.

    B The sides of a polygon overlap.

    C A polygon is formed by 3 or more line segments.

    D The sides of a polygon meet only at its endpoints.

Spiral Review

For Exercises 36 and 37, use the figures at the right.

36. **ALGEBRA** The quadrilaterals are similar. Find the value of x. (Lesson 10-7)
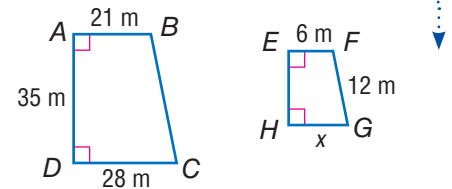
37. **GEOMETRY** Classify figure $ABCD$. (Lesson 10-6)

38. **PROBABILITY** Two students will be randomly selected from a group of seven to present their reports. If Carla and Pedro are in the group of 7, what is the probability that Carla will be selected first and Pedro selected second? (Lesson 9-4)

39. **RUNNING** Use the information in the table to find the rate of change. (Lesson 6-3)

Distance (m)	15	30	45	60
Time (s)	10	20	30	40

Add or subtract. Write each sum or difference in simplest form. (Lesson 5-3)

40. $3\frac{2}{9} + 5\frac{4}{9}$ **41.** $5\frac{1}{3} - 2\frac{1}{6}$ **42.** $1\frac{3}{7} + 6\frac{1}{4}$ **43.** $9\frac{4}{5} - 4\frac{7}{8}$

GET READY for the Next Lesson

PREREQUISITE SKILL Graph and label each point on the same coordinate plane. (Lesson 2-3)

44. $A(-2, 3)$ **45.** $B(4, 3)$ **46.** $C(2, -1)$ **47.** $D(-4, -1)$

Geometry Lab
Tessellations

In this lab, you will create tessellations.

ACTIVITY

STEP 1 Draw a square on the back of an index card. Then draw a triangle on the inside of the square and a trapezoid on the bottom of the square as shown.

STEP 2 Cut out the square. Then cut out the triangle and slide it from the right side of the square to the left side of the square. Cut out the trapezoid and slide it from the bottom to the top of the square.

STEP 3 Tape the figures together to form a pattern.
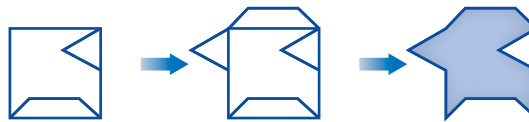
STEP 4 Trace this pattern onto a sheet of paper as shown to create a tessellation.
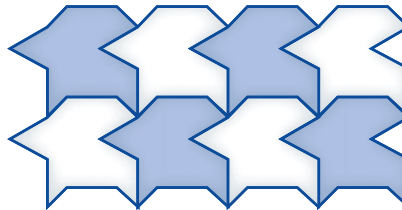
CHECK Your Progress

Create a tessellation using each pattern.

a.

b.

c.
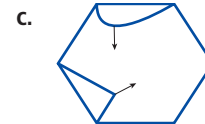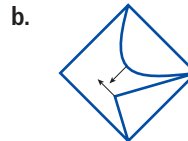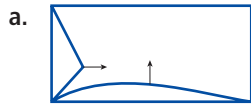
ANALYZE THE RESULTS

1. Design and describe your own tessellation pattern.

2. **MAKE A CONJECTURE** *Congruent figures* have corresponding sides of equal length and corresponding angles of equal measure. Explain how congruent figures are used in your tessellation.

10-9 Translations

MAIN IDEA

Graph translations of polygons on a coordinate plane.

New Vocabulary

transformation
translation
congruent figures

▶ MINI Lab

STEP 1 Trace a parallelogram-shaped pattern block onto a coordinate grid. Label the vertices *ABCD*.
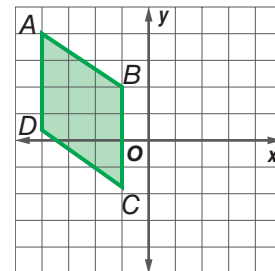
STEP 2 Slide the pattern block over 5 units to the right and 2 units down.

STEP 3 Trace the figure in its new position. Label the vertices *A′*, *B′*, *C′*, and *D′*.
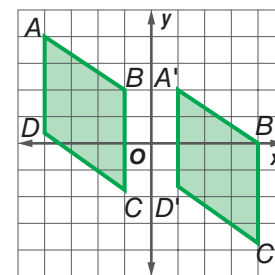
1. Trace the horizontal and vertical path between corresponding vertices. What do you notice?

2. Add 5 to each *x*-coordinate of the vertices of the original figure. Then subtract 2 from each *y*-coordinate of the vertices of the original figure. What do you notice?

A **transformation** maps one figure onto another. When you move the figure without turning it, the motion is called a **translation**.

When translating a figure, every point of the original figure is moved the same distance and in the same direction.

EXAMPLE Graph a Translation

1 **Translate quadrilateral *HIJK* 2 units left and 5 units up. Graph quadrilateral *H′ I′ J′ K′*.**

• Move each vertex of the figure 2 units left and 5 units up. Label the new vertices *H′*, *I′*, *J′*, and *K′*.

• Connect the vertices to draw the trapezoid. The coordinates of the vertices of the new figure are $H'(-4, 1)$, $I'(-3, 4)$, $J'(1, 3)$, and $K'(-1, 1)$.
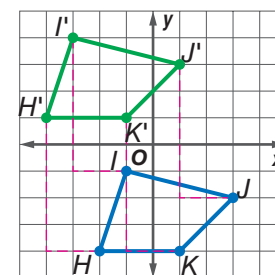
✓ CHECK Your Progress

a. Translate quadrilateral *HIJK* 4 units up and 2 units right. Graph quadrilateral *H′ I′ J′ K′*.

When a figure has been translated, the original figure and the translated figure, or *image*, are congruent. **Congruent figures** have the same size and same shape, and the corresponding sides and angles have equal measures.

$\triangle ABC \cong \triangle A'B'C'$

You can increase or decrease the coordinates of the vertices of a figure by a fixed amount to find the coordinates of the translated vertices.

A *positive* integer describes a translation right or up on a coordinate plane. A *negative* integer describes a translation left or down.
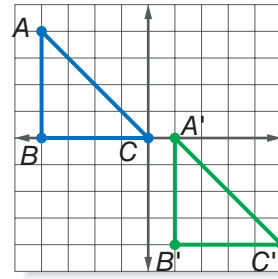
EXAMPLE Find Coordinates of a Translation

2 Triangle *LMN* has vertices $L(-1, -2)$, $M(6, -3)$, and $N(2, -5)$. Find the vertices of $\triangle L'M'N'$ after a translation of 6 units left and 4 units up. Then graph the figure and its translated image.

The vertices can be found by adding -6 to the *x*-coordinates and 4 to the *y*-coordinates.

Add -6 to each *x*-coordinate.   Add 4 to each *y*-coordinate.

Vertices of △LMN	$(x + (-6), y + 4)$	Vertices of △L'M'N'
$L(-1, -2)$	$(-1 + (-6), -2 + 4)$	$L'(-7, 2)$
$M(6, -3)$	$(6 + (-6), -3 + 4)$	$M'(0, 1)$
$N(2, -5)$	$(2 + (-6), -5 + 4)$	$N'(-4, -1)$

Use the vertices of $\triangle LMN$ and of $\triangle L'M'N'$ to graph each triangle.
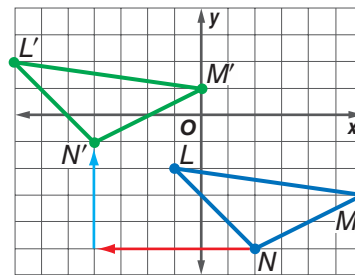
✓ CHECK Your Progress

b. Triangle *TUV* has vertices $T(6, -3)$, $U(-2, 0)$, and $V(-1, 2)$. Find the vertices of $\triangle T'U'V'$ after a translation of 3 units right and 4 units down. Then graph the figure and its translated image.

In Example 2, $\triangle LMN$ was translated 6 units left and 4 units up. This translation can be described using the ordered pair $(-6, 4)$. In Check Your Progress b., $\triangle TUV$ was translated 3 units right and 4 units down. This translation can be described using the ordered pair $(3, -4)$.

Example 1
(p. 553)

1. Translate △ABC 3 units left and 3 units down. Graph △A'B'C'.
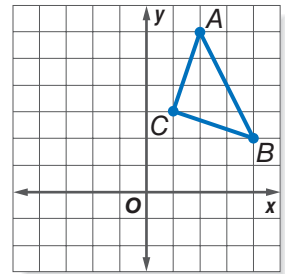
Example 2
(p. 554)

Quadrilateral DEFG has vertices D(1, 0), E(−2, −2), F(2, 4), and G(6, −3). Find the vertices of D'E'F'G' after each translation. Then graph the figure and its translated image.

2. 4 units right, 5 units down   3. 6 units right

4. **MAPS** Nakos explores part of the Denver Zoo in Colorado as shown. He starts at the felines and then visits the hoofed animals. Describe this translation in words and as an ordered pair.

Practice and Problem Solving

5. Translate △HIJ 2 units right and 6 units down. Graph △H'I'J'.
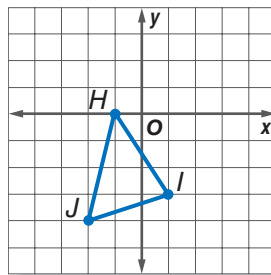
6. Translate rectangle KLMN 1 unit left and 3 units up. Graph rectangle K'L'M'N'.
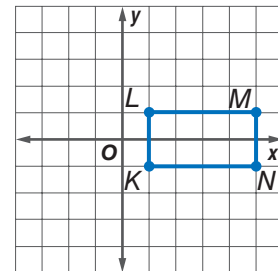
Triangle PQR has vertices P(0, 0), Q(5, −2), and R(−3, 6). Find the vertices of P'Q'R' after each translation. Then graph the figure and its translated image.

7. 6 units right, 5 units up   8. 8 units left, 1 unit down

9. 3 units left   10. 9 units down

GAMES When playing the game shown at the right, the player can move horizontally or vertically across the board. Describe each of the following as a translation in words and as an ordered pair.

11. Green player

12. Orange player
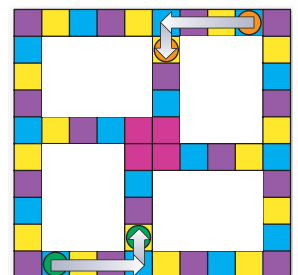
13. Triangle *ABC* is translated 2 units left and 3 units down. Then the translated figure is translated 3 units right. Graph the resulting triangle.

14. Parallelogram *RSTU* is translated 3 units right and 5 units up. Then the translated figure is translated 2 units left. Graph the resulting parallelogram.
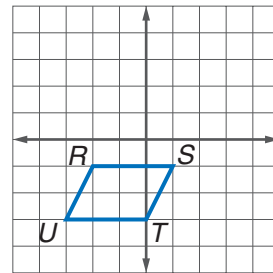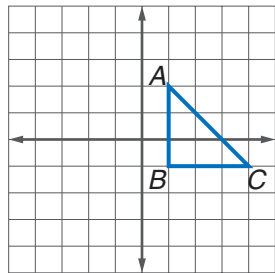
15. ART Marjorie Rice creates art using tessellations. At the right is her artwork of fish. Explain how translations and tessellations were used in the figure.

16. RESEARCH Use the Internet or another source to find other pieces of art that contain tessellations of translations. Describe how the artists incorporated both ideas into their work.

17. Triangle *FGH* has vertices $F(7, 6)$, $G(3, 4)$, $H(1, 5)$. Find the coordinates of $\triangle F'G'H'$ after a translation $1\frac{1}{2}$ units right and $3\frac{1}{2}$ units down. Then graph the figure and its translated image.

REASONING **The coordinates of a point and its image after a translation are given. Describe the translation in words and as an ordered pair.**

18. $N(0, -3) \rightarrow N'(2, 2)$

19. $M(2, 4) \rightarrow M'(-3, 1)$

20. $P(-2, -1) \rightarrow P'(3, -2)$

21. $Q(-4, 0) \rightarrow Q'(1, 4)$

H.O.T. Problems

22. CHALLENGE Is it possible to make a tessellation with translations of an equilateral triangle? Explain your reasoning.

23. Which One Doesn't Belong? Identify the transformation that is not the same as the other three. Explain your reasoning.
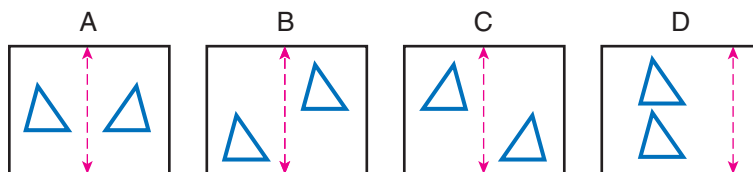
24. **WRITING IN MATH** Triangle *ABC* is translated 4 units right and 2 units down. Then the translated image is translated again 7 units left and 5 units up. Describe the final translated image in words.

25. Which graph shows a translation of the letter Z?

A
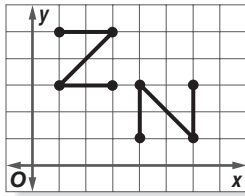
C
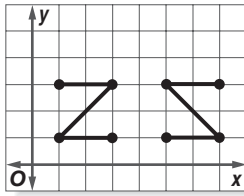
B
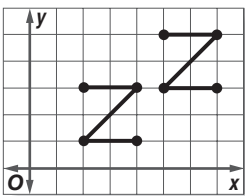
D
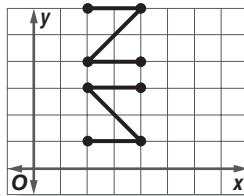
26. If point *A* is translated 4 units left and 3 units up, what will be the coordinates of point *A* in its new position?

F $(4, 4)$

G $(-5, 5)$

H $(-5, -1)$

J $(-4, 3)$
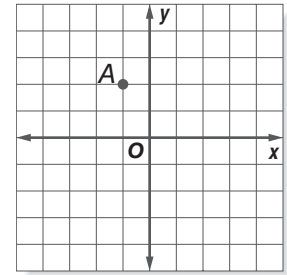
Spiral Review

27. GEOMETRY What is the name of a polygon with eight sides? (Lesson 10-8)

28. GEOMETRY The triangles at the right are similar. What is the measure of $\angle F$? (Lesson 10-7)
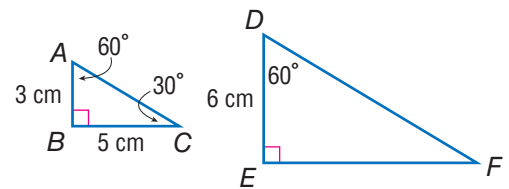
29. FOOD For dinner, you can choose one of two appetizers, one of four entrées, and one of three desserts. How many possible unique dinners can you choose? (Lesson 9-3)

For each set of data, describe how the range would change if the value 15 was added to the data set. (Lesson 8-2)

30. $\{8, 17, 32\}$ **31.** $\{22, 38, 41, 77\}$ **32.** $\{10, 10, 19\}$ **33.** $\{7, 11, 13\}$

Write each percent as a decimal. (Lesson 4-7)

34. 83.8%   **35.** 56.7%   **36.** 3.8%   **37.** 102.6%

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether each figure can be folded in half so that one side matches the other. Write *yes* or *no*.

38.   **39.**   **40.**   **41.**

10-10 Reflections

MAIN IDEA

Identify figures with line symmetry and graph reflections on a coordinate plane.

New Vocabulary

line symmetry
line of symmetry
reflection
line of reflection

Math Online

glencoe.com
• Extra Examples
• Personal Tutor
• Self-Check Quiz

▷ MINI Lab

VISION Scientists have determined that the human eye uses symmetry to see. It is possible to understand what you are looking at, even if you do not see all of it.

DOX
CODE

1. The top half of the words at the right are missing. Identify the words.

2. List all the capital letters of the alphabet that, when folded across a horizontal line, look exactly the same.

3. On a piece of paper, write the bottom half of other words that, when reflected across a horizontal line, look exactly the same.

Figures that match exactly when folded in half have **line symmetry**. The figures at the right have line symmetry.
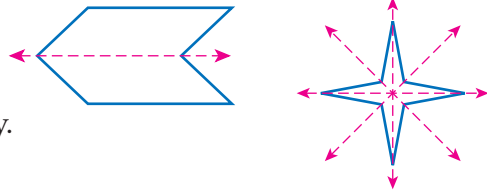
Each fold line is called a **line of symmetry**.

Real-World EXAMPLES

GRAPHIC DESIGN Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

1 E

2 G   no symmetry

3 H

✓ CHECK Your Progress

a.

b.

c.
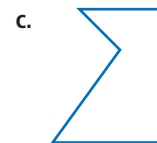
A **reflection** is a mirror image of the original figure. It is the result of a transformation of a figure over a line called a **line of reflection**.

EXAMPLE   **Reflect a Figure Over the *x*-axis**

4   **Triangle *ABC* has vertices *A*(5, 2), *B*(1, 3), and *C*(−1, 1). Graph the figure and its reflected image over the *x*-axis. Then find the coordinates of the vertices of the reflected image.**

The *x*-axis is the line of reflection. So, plot each vertex of *A′B′C′* the same distance from the *x*-axis as its corresponding vertex on *ABC*.
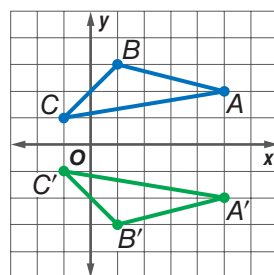
Point *A* is 2 units above the *x*-axis, …

… so point *A′* is plotted 2 units below the *x*-axis.

The coordinates are *A′*(5, −2), *B′*(1, −3), and *C′*(−1, −1).

CHECK Your Progress

d. Rectangle *GHIJ* has vertices *G*(3, −4), *H*(3, −1), *I*(−2, −1), and *J*(−2, −4). Graph the figure and its image after a reflection over the *x*-axis. Then find the coordinates of the reflected image.

EXAMPLE   **Reflect a Figure Over the *y*-axis**

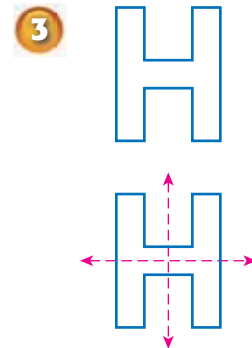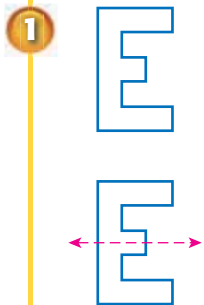**5**   **Quadrilateral *KLMN* has vertices *K*(2, 3), *L*(5, 1), *M*(4, −2), and *N*(1, −1). Graph the figure and its reflected image over the *y*-axis. Then find the coordinates of the vertices of the reflected image.**

The *y*-axis is the line of reflection. So, plot each vertex of *K′L′M′N′* the same distance from the *y*-axis as its corresponding vertex on *KLMN*.

Point *K′* is 2 units to the left of the *y*-axis.
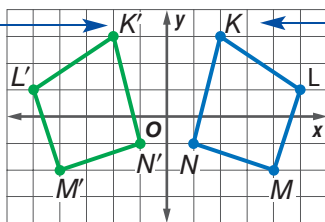
Point *K* is 2 units to the right of the *y*-axis.

The coordinates are *K′*(−2, 3), *L′*(−5, 1), *M′*(−4, −2), and *N′*(−1, −1).

CHECK Your Progress

e. Triangle *PQR* has vertices *P*(1, 5), *Q*(3, 7), and *R*(5, −1). Graph the figure and its reflection over the *y*-axis. Then find the coordinates of the reflected image.

✔ CHECK Your Understanding

Examples 1–3
(p. 558)

Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

1.

2.
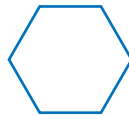
3.

4. **INSECTS** Identify the number of lines of symmetry in the photo of the butterfly at the right.

Example 4
(p. 559)

Graph each figure and its reflection over the x-axis. Then find the coordinates of the reflected image.

5. $\triangle ABC$ with vertices $A(5, 8)$, $B(1, 2)$, and $C(6, 4)$

6. quadrilateral $DEFG$ with vertices $D(-4, 6)$, $E(-2, -3)$, $F(2, 2)$, and $G(4, 9)$

Example 5
(p. 559)

Graph each figure and its reflection over the y-axis. Then find the coordinates of the reflected image.

7. $\triangle QRS$ with vertices $Q(2, -5)$, $R(5, -5)$, and $S(2, 3)$

8. parallelogram $WXYZ$ with vertices $W(-4, -2)$, $X(-4, 3)$, $Y(-2, 4)$, and $Z(-2, -1)$

▶ Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
9–14 23–24	1, 3
15–18	4
19–22	5

Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

9.

10.
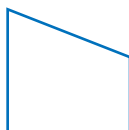
11.

12.

13.
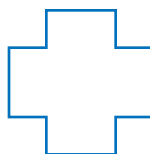
14.
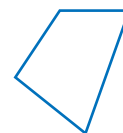
Graph each figure and its reflection over the x-axis. Then find the coordinates of the reflected image.

15. TUV with vertices $T(-6, -1)$, $U(-2, -3)$, and $V(5, -4)$

16. MNP with vertices $M(2, 1)$, $N(-3, 1)$, and $P(-1, 4)$

17. square $ABCD$ with vertices $A(2, 4)$, $B(-2, 4)$, $C(-2, 8)$, and $D(2, 8)$

18. $WXYZ$ with vertices $W(-1, -1)$, $X(4, 1)$, $Y(4, 5)$, and $Z(1, 7)$

Graph each figure and its reflection over the *y*-axis. Then find the coordinates of the reflected image.

19. $\triangle RST$ with vertices $R(-5, 3)$, $S(-4, -2)$, and $T(-2, 3)$

20. $\triangle GHJ$ with vertices $G(4, 2)$, $H(3, -4)$, and $J(1, 1)$

21. parallelogram $HIJK$ with vertices $H(-1, 3)$, $I(-1, -1)$, $J(2, -2)$, and $K(2, 2)$

22. quadrilateral $DEFG$ with vertices $D(1, 0)$, $E(1, -5)$, $F(4, -1)$, and $G(3, 2)$

23. GATES Describe the location of the line(s) of symmetry in the photograph of Brandenburg Gate in Berlin, Germany.

24. FLAGS Flags of some countries have line symmetry. Of the flags shown below, which flags have line symmetry? Copy and draw all lines of symmetry.
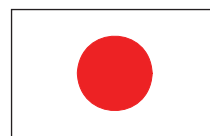
Nigeria	**Ghana**	**Japan**	**Mexico**

25. MUSIC Use the photo at the left to determine how many lines of symmetry the body of a violin has.

For Exercises 26–29, use the graph shown at the right.

26. Identify the pair(s) of figures for which the *x*-axis is the line of reflection.

27. For which pair(s) of figures is the line of reflection the *y*-axis?

28. What type of transformation do figures B and C represent?

29. Describe the possible transformation(s) required to move figure A onto figure D.

30. $\triangle RST$ is reflected over the *x*-axis and then translated 3 units to the left and 2 units down. Graph the resulting triangle.

31. $\triangle MNP$ is translated 2 units right and 3 units up. Then the translated figure is reflected over the *y*-axis. Graph the resulting triangle.
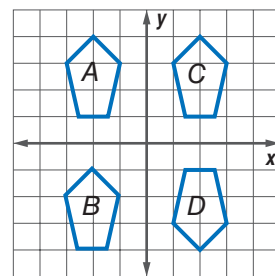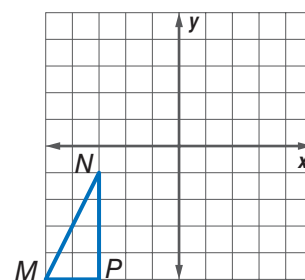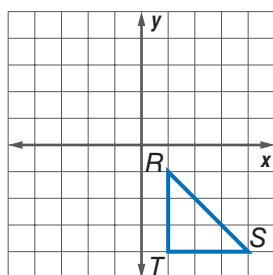
The coordinates of a point and its image after a reflection are given. Describe the reflection as over the *x*-axis or *y*-axis.

32. $A(-3, 5) \rightarrow A'(3, 5)$

33. $M(3, 3) \rightarrow M'(3, -3)$

34. $X(-1, -4) \rightarrow X'(-1, 4)$

35. $W(-4, 0) \rightarrow W'(4, 0)$

H.O.T. Problems

36. OPEN ENDED Make a tessellation using a combination of translations and reflections of polygons. Explain your method.

37. CHALLENGE Triangle *JKL* has vertices $J(-7, 4)$, $K(7, 1)$, and $L(2, -2)$. Without graphing, find the new coordinates of the vertices of the triangle after a reflection first over the *x*-axis and then over the *y*-axis.

38. WRITING IN MATH Draw a figure on a coordinate plane and its reflection over the *y*-axis. Explain how the *x*- and *y*-coordinates of the reflected figure relate to the *x*- and *y*-coordinates of the original figure. Then repeat, this time reflecting the figure over the *x*-axis.

TEST PRACTICE

39. The figure shown was transformed from quadrant II to quadrant III.
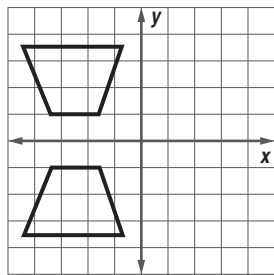
This transformation best represents which of the following?

A translation   **C** reflection

B tessellation   **D** rotation

40. If *ABCD* is reflected over the *x*-axis and translated 5 units to the right, which is the resulting image of point *B*?
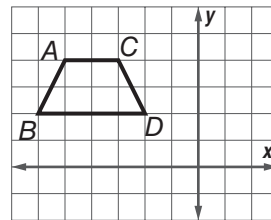
F $(-1, -2)$ **H** $(-1, 2)$

G $(-11, 2)$ **J** $(11, 2)$

Spiral Review

41. GEOMETRY Triangle *FGH* has vertices $F(-3, 7)$, $G(-1, 5)$, and $H(-2, 2)$. Graph the figure and its image after a translation 4 units right and 1 unit down. Write the ordered pairs for the vertices of the image. (Lesson 10-9)

42. GEOMETRY Melissa wishes to construct a tessellation for a wall hanging made only from regular decagons. Is this possible? Explain. (Lesson 10-8)

Estimate. (Lesson 5-1)

43. $\frac{4}{9} + 8\frac{1}{9}$

44. $\frac{1}{9} \times \frac{2}{5}$

45. $12\frac{1}{4} \div 5\frac{6}{7}$

CHAPTER
10
Study Guide
and Review

Math Online   glencoe.com
• STUDY TO GO
• Vocabulary Review

FOLDABLES Study Organizer ▶ GET READY to Study

Be sure the following Big Ideas are noted in your Foldable.

(Foldable diagram: What I Know About Polygons | What I Need to Know | What I've Learned)

BIG Ideas

Angles (Lessons 10-1 and 10-2)
• Two angles are adjacent if they have the same vertex, share a common side, and do not overlap.
• Two angles are vertical if they are opposite angles formed by the intersection of two lines.
• Two angles are complementary if the sum of their measures is 90°.
• Two angles are supplementary if the sum of their measures is 180°.

Triangles (Lesson 10-4)
• The sum of the measures of the angles of a triangle is 180°.

Quadrilaterals (Lesson 10-6)
• The sum of the measures of the angles of a quadrilateral is 360°.

Similar Figures (Lesson 10-7)
• If two figures are similar then the corresponding sides are proportional and the corresponding angles are congruent.

Transformations (Lessons 10-9 and 10-10)
• When translating a figure, every point in the original figure is moved the same distance in the same direction.
• When reflecting a figure, every point in the original figure is the same distance from the line of reflection as its corresponding point on the original figure.

Key Vocabulary

acute angle (p. 511)
angle (p. 510)
circle graph (p. 518)
complementary angles (p. 514)
congruent angles (p. 511)
congruent figures (p. 554)
degrees (p. 510)
hexagon (p. 546)
indirect measurement (p. 542)
line of symmetry (p. 558)
line symmetry (p. 558)
obtuse angle (p. 511)
octagon (p. 546)
parallelogram (p. 533)

pentagon (p. 546)
polygon (p. 546)
quadrilateral (p. 533)
reflection (p. 559)
regular polygon (p. 546)
rhombus (p. 533)
right angle (p. 511)
similar figures (p. 540)
straight angle (p. 511)
supplementary angles (p. 514)
tessellation (p. 548)
transformation (p. 553)
translation (p. 553)
trapezoid (p. 533)
triangle (p. 524)
vertex (p. 510)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. Two angles with measures adding to 180° are called <u>complementary angles</u>.

2. A <u>hexagon</u> is a polygon with 6 sides.

3. An angle with a measure of less than 90° is called a <u>right angle</u>.

4. The <u>vertex</u> is where the sides of an angle meet.

5. The point (3, −2) when translated up 3 units and to the left 5 units becomes (<u>6, −7</u>).

6. A <u>trapezoid</u> has both pairs of opposite sides parallel.

Lesson-by-Lesson Review

10-1 Angle Relationships (pp. 510–513)

For Exercises 7 and 8, refer to the figure at the right to identify each pair of angles. Justify your response.
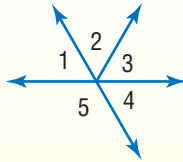
7. a pair of vertical angles

8. a pair of adjacent angles

Example 1 Refer to the figure below. Identify a pair of vertical angles.

∠1 and ∠4 are opposite angles formed by the intersection of two lines.
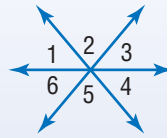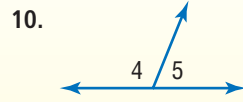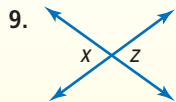
∠1 and ∠4 are vertical angles.

10-2 Complementary and Supplementary Angles (pp. 514–517)

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

9.

10.

Example 2 Find the value of x.

$$x + 27 = 90$$
$$\underline{-27 = -27}$$
$$x = 63$$
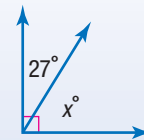
10-3 Statistics: Display Data in a Circle Graph (pp. 518–523)

11. **COLORS** The table shows favorite shades of blue. Display the set of data in a circle graph.

Shade	Percent
Navy	35%
Sky/Light Blue	30%
Aquamarine	17%
Other	18%

Example 3 Which pizza was chosen by about twice as many people as supreme?

Pepperoni was chosen by about twice as many people as supreme.

Types of Pizza

Vegetable — 15%
Cheese — 18%
Supreme — 22%
Pepperoni — 45%
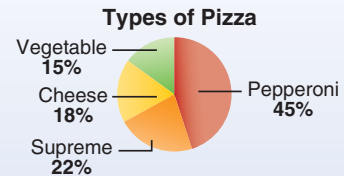
10-4 Triangles (pp. 524–529)

ALGEBRA Find the value of x.

12.

13.

Example 4 Find the value of x.

$$x + 64 + 67 = 180$$
$$x + 131 = 180$$
$$\underline{-131 = -131}$$
$$x = 49$$

10-5 **PSI: Logical Reasoning** (pp. 530–531)

14. SPORTS Donnie, Jenna, Milo, and
Barbara play volleyball, field hockey,
golf, and soccer but not in that order.
Use the clues given below to find the
sport each person plays.

- Donnie does not like golf, volleyball,
or soccer.
- Neither Milo nor Jenna likes golf.
- Milo does not like soccer.

15. FOOD Angelo's Pizza Parlor makes
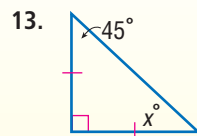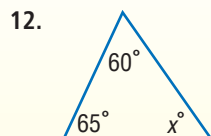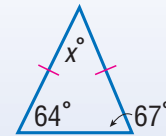square pizzas. After baking, the pizzas
are cut along one diagonal into two
triangles. Classify the triangles made.

Example 5 Todd, Virginia, Elaine, and
Peter are siblings. Todd was born after
Peter, but before Virginia. Elaine is the
oldest. Who is the youngest in the family?

Use logical reasoning to determine the
youngest of the family.

You know that Elaine is the oldest, so she
is first on the list. Todd was born after
Peter, but before Virginia. So, Peter was
second and then Todd was born. Virginia
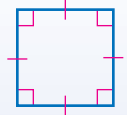is the youngest of the family.

10-6 **Quadrilaterals** (pp. 533–538)

Classify the quadrilateral with the name
that best describes it.

16.

17.
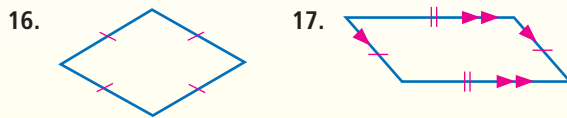
18. GEOMETRY What quadrilateral does not
have opposite sides congruent?

Example 6 Classify the
quadrilateral using the
name that *best* describes it.

The quadrilateral is a parallelogram
with 4 right angles and 4 congruent sides.
It is a square.

10-7 **Similar Figures** (pp. 540–545)

Find the value of x in each pair of similar
figures.

19. 5 cm 8 cm   **20.** 4 m   6 m
16 cm   7 m x
x
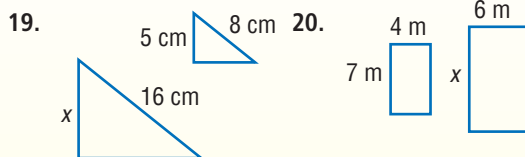
21. FLAGPOLES Hiroshi is 1.6 meters tall
and casts a shadow 0.53 meter in
length. How tall is a flagpole if it casts
a shadow 2.65 meters in length?

Example 7

Find the value of x
in the pair of
similar figures.
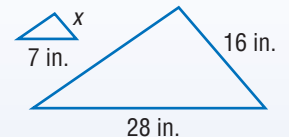
x 7 in.   16 in.
      28 in.

$\dfrac{7}{28} = \dfrac{x}{16}$ Write a proportion.

$28 \cdot x = 7 \cdot 16$ Find the cross products.

$28x = 112$ Simplify.

$\dfrac{28x}{28} = \dfrac{112}{28}$ Divide each side by 28.

$x = 4$ Simplify.

So, the value of x is 4.

10-8 Polygons and Tessellations (pp. 546–551)

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

22.

23.
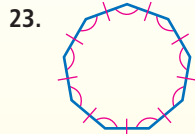
24. **ALGEBRA** Find the measure of each angle of a regular 12-gon.

Example 8 Determine whether the figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.
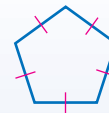
Since the polygon has 5 congruent sides and 5 congruent angles, it is a regular pentagon.

10-9 Translations (pp. 553–557)

Triangle PQR has coordinates $P(4, -2)$, $Q(-2, -3)$, and $R(-1, 6)$. Find the coordinates of $P'Q'R'$ after each translation. Then graph each translation.

25. 6 units left, 3 units up

26. 4 units right, 1 unit down

27. 3 units left

28. 7 units down

Example 9 Find the coordinates of $\triangle G'H'I'$ after a translation of 2 units left and 4 units up.

The vertices of $\triangle G'H'I'$ are $G'(-2, 7)$, $H'(2, 3)$, and $I'(-4, 1)$.
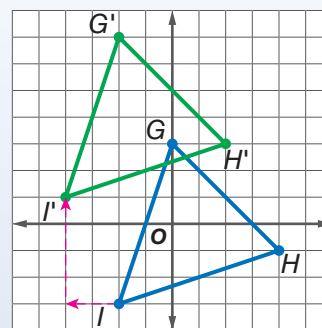
10-10 Reflections (pp. 558–562)

Find the coordinates of each figure after a reflection over the given axis. Then graph the figure and its reflected image.

29. $\triangle RST$ with coordinates $R(-1, 3)$, $S(2, 6)$, and $T(6, 1)$; x-axis

30. parallelogram $ABCD$ with coordinates $A(1, 3)$, $B(2, -1)$, $C(5, -1)$, and $D(4, 3)$; y-axis

31. rectangle $EFGH$ with coordinates $E(4, 2)$, $F(-2, 2)$, $G(-2, 5)$, and $H(4, 5)$; x-axis

Example 10 Find the coordinates of $\triangle C'D'E'$ after a reflection over the y-axis. Then graph its reflected image.

The vertices of $\triangle C'D'E'$ are $C'(-3, 4)$, $D'(-2, 1)$, and $E'(-5, 3)$.
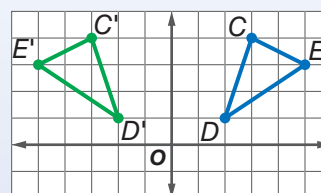
Name each angle in four ways. Then classify each angle as *acute*, *obtuse*, *right*, or *straight*.
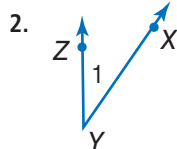
1.

2.

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

3. 125° 25°

4. 45° 45°

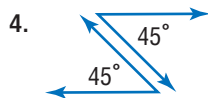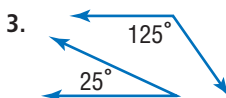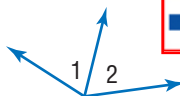5. **GEOMETRY** Classify the angle pair at the right as *vertical*, *adjacent*, or *neither*.

6. **MULTIPLE CHOICE** The table shows the results of a survey. The results are to be displayed in a circle graph. Which statement about the graph is *not* true?

| Favorite Type of Bagels	
Type	**Students**
blueberry	8
cinnamon raisin	9
everything	18
plain	32

A About 12% of students chose blueberry as their favorite bagel.
B The blueberry section on the graph will have an angle measure of about 43°.
C The everything and plain sections on the circle graph form supplementary angles.
D Plain bagels were preferred more than any other type of bagel.

ALGEBRA **Find the missing measure in each triangle with the given angle measures.**

7. $75°, 25.5°, x°$

8. $23.5°, x°, 109.5°$

9. **ALGEBRA** Numbers ending in zero or five are divisible by five. Are the numbers 25, 893, and 690 divisible by 5? Use the *logical reasoning* strategy.

ALGEBRA **Find the value of x in each quadrilateral.**

10.

84° 92° 122° $x°$

11.

$x°$ 70° 58°
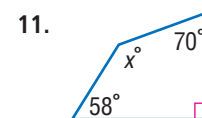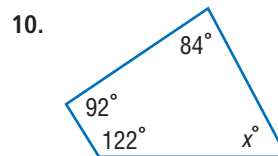
12. **ART** A drawing is enlarged so that it is 14 inches long and 11 inches wide. If the original length of the drawing is 8 inches, what is its width?

13. **GEOMETRY** Can a regular heptagon, with angle measures that total 900°, be used by itself to make a tessellation? Explain.

14. **MULTIPLE CHOICE** Which quadrilateral does *not* have opposite sides congruent?

F parallelogram   **H** trapezoid
G square   **J** rectangle

15. **ALGEBRA** Square *ABCD* is shown. What are the vertices of *A'B'C'D'* after a translation 2 units right and 2 units down? Graph the translated image.
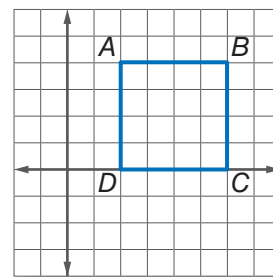
16. **GEOMETRY** Draw a figure with one line of symmetry. Then draw a figure with no lines of symmetry.

PART 1 Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following two angles are complementary?

A ∠RXS and ∠TXU

B ∠SXT and ∠TXU

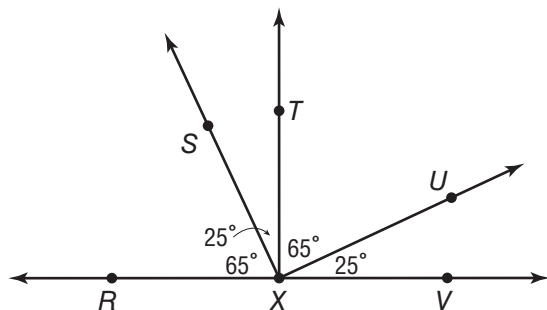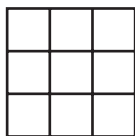C ∠RXS and ∠SXV

D ∠SXR and ∠SXV

2. A square is divided into 9 congruent squares. Which of the following methods can be used to find the area of the larger square, given the area of one of the smaller squares?

F Multiply the area of the larger square by 9.

G Add 9 to the area of one of the smaller squares.

H Multiply the area of one of the smaller squares by 9.

J Add the area of the larger square to the sum of the areas of each of the 9 smaller squares.

3. Which of the following groups does *not* contain equivalent fractions, decimals, and percents?

A $\frac{9}{20}$, 0.45, 45%

B $\frac{3}{10}$, 0.3, 30%

C $\frac{7}{8}$, 0.875, 87.5%

D $\frac{1}{100}$, 0.1, 1%

4. The table below shows all the possible outcomes when tossing two fair coins at the same time.

1st Coin	2nd Coin
H	H
H	T
T	H
T	T

Which of the following must be true?

F The probability that both coins have the same outcome is $\frac{1}{4}$.

G The probability of getting at least one tail is higher than the probability of getting two heads.

H The probability that exactly one coin will turn up heads is $\frac{3}{4}$.

J The probability of getting at least one tail is lower than the probability of getting two tails.

5. Seth has $858.60 in his savings account. He plans to spend 15% of his savings on a bicycle. Which of the following represents the amount Seth plans to spend on the bicycle?

A $182.79   C $128.79

B $171.72   D $122.79

6. A manager took an employee to lunch. If the lunch was $48 and she left a 20% tip, how much money did she spend on lunch?

   F $68.00   **H** $55.80

   G $57.60   **J** $38.40

7. What is the measure of ∠1 in the figure below?
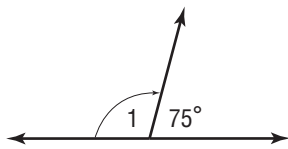
   A 15°   **C** 100°

   B 25°   **D** 105°

8. Josiah found the mean and median of the following list of numbers.

   11, 17, 17

   If the number 25 is added to this list, then which of the following statements would be true?

   F The mean would increase.

   G The mean would decrease.

   H The median would increase.

   J The median would decrease.

TEST-TAKING TIP

Question 8 Sometimes, it is not necessary to perform any calculations in order to answer the question correctly. In Question 8, you can use number sense to eliminate certain answer choices. Not having to perform calculations can help save time during a test.

PART 2   Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Three students are to be chosen from 8 auditions to star in the school play. In how many ways can these 3 students be chosen?

PART 3   Extended Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper. Show your work.

10. Use triangle *XYZ* to answer the following questions.
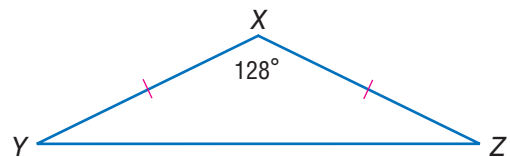
   a. Classify angle *X*.

   b. Classify angle *Y*.

   c. Classify the triangle by its sides and angles.

   d. If ∠*Y* is congruent to ∠*Z*, find the measure of ∠*Z*. Explain.

   e. Can triangle *XYZ* be used by itself to make a tesselation? If so, include a drawing of the tesselation. If not, explain why not.

NEED EXTRA HELP?										
If You Missed Question...	1	2	3	4	5	6	7	8	9	10
Go to Lesson...	10-1	10-5	6-8	9-1	7-1	7-4	10-2	8-2	9-5	10-5