

# CHAPTER 2

# Integers

## BIG Idea

- Add, subtract, multiply, or divide integers to solve problems and justify solutions.

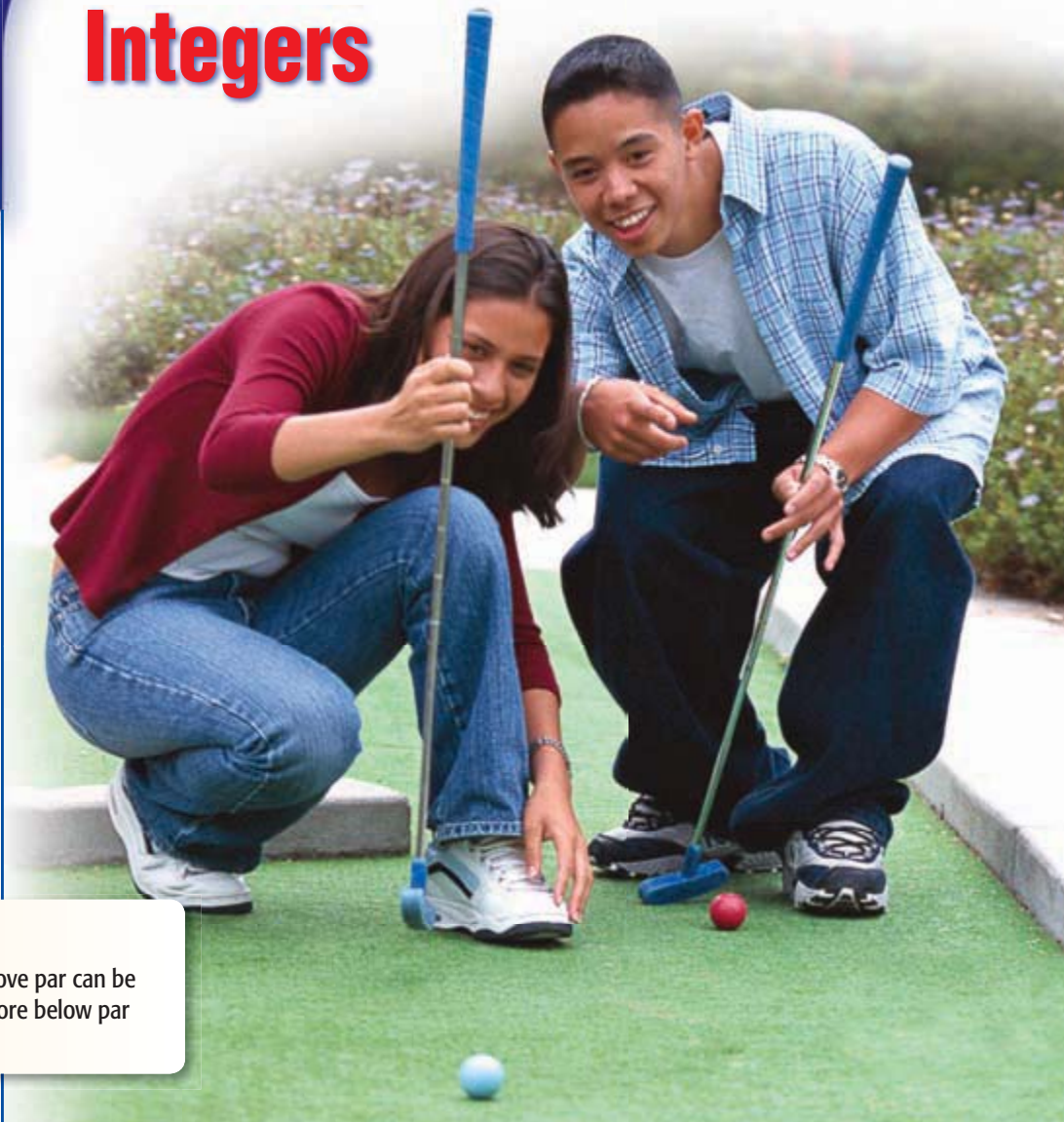
## Key Vocabulary

**graph** (p. 80)

**integer** (p. 80)

**negative integer** (p. 80)

**positive integer** (p. 80)



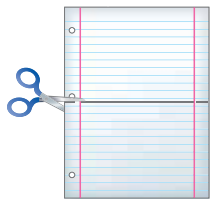
## Real-World Link

**Sports** In miniature golf, a score above par can be written as a positive integer and a score below par can be written as a negative integer.

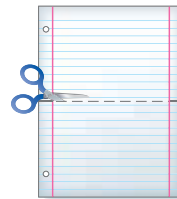
## FOLDABLES<sup>®</sup> Study Organizer

**Integers** Make this Foldable to help you organize your notes. Begin with two sheets of  $8\frac{1}{2}$ " by 11" paper.

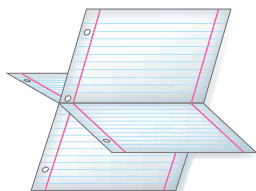
- 1** **Fold** one sheet in half from top to bottom. Cut along fold from edges to margin.



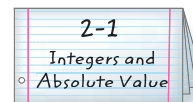
- 2** **Fold** the other sheet in half from top to bottom. Cut along fold between margins.



- 3** **Insert** first sheet through second sheet and align folds.



- 4** **Label** each inside page with a lesson number and title.



# GET READY for Chapter 2

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2

**Math Online**

Take the Online Readiness Quiz at [glencoe.com](http://glencoe.com).

## Option 1

Take the Quick Check below. Refer to the Quick Review for help.

### QUICK Quiz

Replace each ● with  $<$  or  $>$  to make a true sentence. (Prior Grade)

1. 1,458 ● 1,548    2. 36 ● 34  
3. 1.02 ● 1.20    4. 76.7 ● 77.6

5. **COINS** Philippe has \$5.17 in coins and Garrett has \$5.71 in coins. Who has the greater amount? (Prior Grade)

Evaluate each expression if  $a = 7$ ,  $b = 2$ , and  $c = 11$ . (Prior Grade)

6.  $a + 8$                       7.  $a + b + c$   
8.  $c - b$                       9.  $a - b + 4$

10. **TEMPERATURE** At 8 A.M., it was  $63^{\circ}\text{F}$ . By noon, the temperature had risen 9 degrees Fahrenheit. What was the temperature at noon? (Prior Grade)

Evaluate each expression if  $m = 9$  and  $n = 4$ . (Prior Grade)

11.  $6mn$                       12.  $n \div 2 - 1$   
13.  $m + 5 \times n$           14.  $m^2 \div (n + 5)$

15. **PLANES** The distance in miles that an airplane travels is given by  $rt$  where  $r$  is the rate of travel and  $t$  is the time. Find the distance an airplane traveled if  $t = 4$  hours and  $r = 475$  miles per hour. (Prior Grade)

### QUICK Review

#### Example 1

Replace the ● with  $<$  or  $>$  to make a true sentence.

$$3.14 \bullet 3.41$$

$$3.14$$

$$3.41$$



Line up the decimal points. Starting at the left, compare the digits in each place-value position.

The digits in the tenths place are not the same. Since  $1 \text{ tenth} < 4 \text{ tenths}$ ,  $3.14 < 3.41$ .

#### Example 2

Evaluate the expression  $11 - a + b$  if  $a = 2$  and  $b = 8$ .

$$11 - a + b = 11 - 2 + 8$$

$$= 9 + 8$$

$$= 17$$

Replace  $a$  with 2 and  $b$  with 8.  
Subtract 2 from 11.  
Add 9 and 8.

#### Example 3

Evaluate the expression  $n^2 \div 16 + m$  if  $m = 3$  and  $n = 8$ .

$$n^2 \div 16 + m = 8^2 \div 16 + 3$$

$$= 64 \div 16 + 3$$

$$= 4 + 3$$

$$= 7$$

Replace  $m$  with 3 and  $n$  with 8.

Evaluate  $8^2$ .

Divide 64 by 16.

Add 4 and 3.

# 2-1

# Integers and Absolute Value

## MAIN IDEA

Read and write integers, and find the absolute value of a number.

## New Vocabulary

**integer**  
**negative integer**  
**positive integer**  
**graph**  
**absolute value**

## Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

## ▶ GET READY for the Lesson

**SKATEBOARDING** The bottom of a skateboarding ramp is 8 feet below streetlevel. A value of  $-8$  represents 8 feet *below* street level.

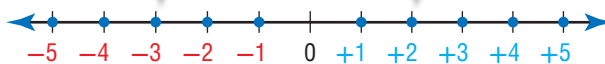
1. What does a value of  $-10$  represent?
2. The top deck of the ramp is 5 feet *above* street level. How can you represent 5 feet *above* street level?



Numbers like 5 and  $-8$  are called integers. An **integer** is any number from the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$  where  $\dots$  means *continues without end*.

**Negative integers** are integers less than zero. They are written with a  $-$  sign.

**Positive integers** are integers greater than zero. They can be written with or without a  $+$  sign.



Zero is neither negative nor positive.

## Real-World EXAMPLES

**WEATHER** Write an integer for each situation.

- 1 an average temperature of 5 degrees below normal  
Because it represents *below* normal, the integer is  $-5$ .
- 2 an average rainfall of 5 inches above normal  
Because it represents *above* normal, the integer is  $+5$  or 5.

## ✓ CHECK Your Progress

Write an integer for each situation.

- a. 6 degrees above normal
- b. 2 inches below normal

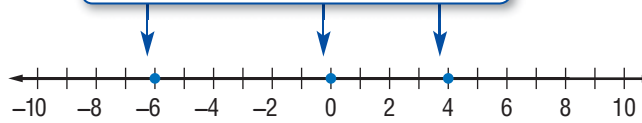
Integers can be graphed on a number line. To **graph** a point on the number line, draw a point on the line at its location.



## EXAMPLE Graph Integers

- 3 Graph the set of integers  $\{4, -6, 0\}$  on a number line.

Draw a number line. Then draw a dot at the location of each integer.



## CHECK Your Progress

Graph each set of integers on a number line.

c.  $\{-2, 8, -7\}$

d.  $\{-4, 10, -3, 7\}$

On the number line below, notice that  $-5$  and  $5$  are each 5 units from 0, even though they are on opposite sides of 0. Numbers that are the same distance from zero on a number line have the same **absolute value**.

## Reading Math

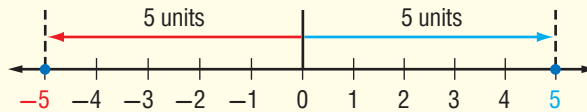
### Absolute Value

$|-5|$  absolute value of negative five

## Absolute Value

Key Concept

**Words** The absolute value of a number is the distance between the number and zero on a number line.



**Examples**

$$|-5| = 5$$

$$|5| = 5$$

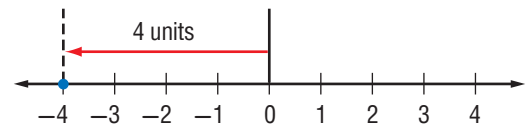
## EXAMPLES Evaluate Expressions

Evaluate each expression.

4  $|-4|$

On the number line, the point  $-4$  is 4 units from 0.

So,  $|-4| = 4$ .



## Study Tip

**Order of Operations**  
The absolute value bars are considered to be a grouping symbol. When evaluating  $|-5| - |2|$ , evaluate the absolute values before subtracting.

5  $|-5| - |2|$

$$|-5| - |2| = 5 - 2 \quad |-5| = 5, |2| = 2$$

So,  $|-5| - |2| = 3$ .

## CHECK Your Progress

Evaluate each expression.

e.  $|8|$

f.  $2 + |-3|$

g.  $|-6| - 5$



## CHECK Your Understanding

**Examples 1, 2**  
(p. 80)

Write an integer for each situation.

- a loss of 11 yards
- 6°F below zero
- a deposit of \$16
- 250 meters above sea level
- FOOTBALL** The quarterback lost 15 yards on one play. Write an integer to represent the number of yards lost.

**Example 3**  
(p. 81)

Graph each set of integers on a number line.

- {11, -5, -8}
- {2, -1, -9, 1}

**Examples 4, 5**  
(p. 81)

Evaluate each expression.

- $|-9|$
- $1 + |7|$
- $|-1| - |-6|$

## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
11–20	1, 2
21–24	3
25–30	4, 5

Write an integer for each situation.

- a profit of \$9
- a bank withdrawal of \$50
- 53°C below zero
- 7 inches more than normal
- 2 feet below flood level
- 160 feet above sea level
- an elevator goes up 12 floors
- no gains or losses on first down
- GOLF** In golf, scores are often written in relationship to *par*, the average score for a round at a certain course. Write an integer to represent a score that is 7 under par.
- PETS** Javier's pet guinea pig gained 8 ounces in one month. Write an integer to describe the amount of weight his pet gained.

Graph each set of integers on a number line.

- {0, 1, -3}
- {3, -7, 6}
- {-5, -1, 10, -9}
- {-2, -4, -6, -8}

Evaluate each expression.

- $|10|$
- $|-12|$
- $|-7| - 5$
- $7 + |4|$
- $|-9| + |-5|$
- $|8| - |-10|$
- $|-10| \div 2 \times |5|$
- $12 - |-8| + 7$
- $|27| \div 3 - |-4|$

- SCUBA DIVING** One diver descended 10 feet, and another ascended 8 feet. Which situation has the greater absolute value? Explain.

### EXTRA PRACTICE

See pages 671, 705.

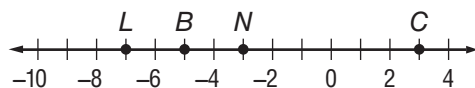
- SCIENCE** If you rub a balloon through your hair, you can make the balloon stick to a wall. Suppose there are 17 positive charges on the wall and 25 negative charges on the balloon. Write an integer for each charge.

### H.O.T. Problems

36. **REASONING** If  $|x| = 3$ , what is the value of  $x$ ?
37. **CHALLENGE** Determine whether the following statement is *true* or *false*. If *false*, give a counterexample.  
*The absolute value of every integer is positive.*
38. **WRITING IN MATH** Write a real-world situation that uses negative integers. Explain what the negative integer means in that situation.

### TEST PRACTICE

39. Which point has a coordinate with the greatest absolute value?



- A Point B  
 B Point C  
 C Point L  
 D Point N

40. Which statement about these real-world situations is *not* true?

- F A \$100 check deposited in a bank can be represented by +100.  
 G A loss of 15 yards in a football game can be represented by -15.  
 H A temperature of 20 below zero can be represented by -20.  
 J A submarine diving 300 feet under water can be represented by +300.

### Spiral Review

Copy and complete each function table. Identify the domain and range. (Lesson 1-10)

41.  $y = x - 4$

$x$	$x - 4$	$y$
4		
5		
6		
7		

42.  $y = 9x$

$x$	$9x$	$y$
0		
1		
2		
3		

43.  $y = 5x + 1$

$x$	$5x + 1$	$y$
1		
2		
3		
4		

44. **GEOMETRY** The table shows the side length and perimeter of several equilateral triangles. Write an expression that describes the perimeter if  $x$  represents the side length. (Lesson 1-9)

Side Length (in.)	2	3	4	5	6
Perimeter (in.)	6	9	12	15	18

### GET READY for the Next Lesson

**PREREQUISITE SKILL** Replace each  $\bullet$  with  $<$  or  $>$  to make a true sentence.

45.  $16 \bullet 6$

46.  $101 \bullet 111$

47.  $87.3 \bullet 83.7$

48.  $1,051 \bullet 1,015$

# 2-2

## Comparing and Ordering Integers

### MAIN IDEA

Compare and order integers.

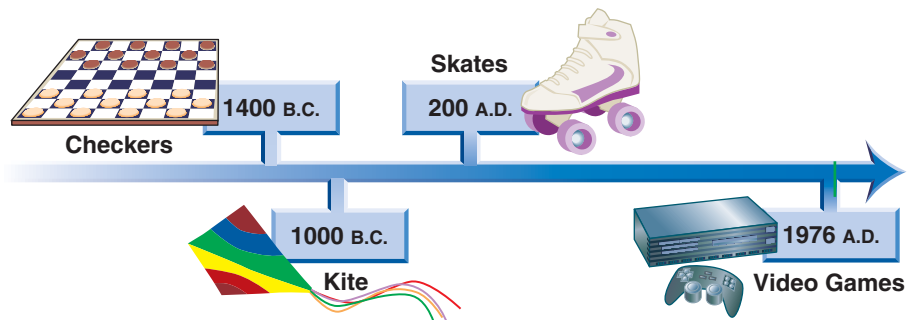
### Math Online

[glencoe.com](http://glencoe.com)

- Concepts In Motion
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Reading in the Content Area

### ▶ GET READY for the Lesson

**TOYS** The timeline shows when some toys were invented.



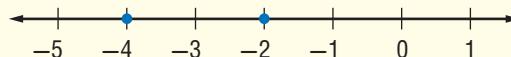
1. The yo-yo was invented around 500 B.C. Was it invented before or after the kite?
2. Modern chess was invented around 600 A.D. Between which two toys was this invented?

When two numbers are graphed on a number line, the number to the left is always less than the number to the right. The number to the right is always greater than the number to the left.

### Compare Integers

Key Concept

**Model**



**Words**

-4 is less than -2.

-2 is greater than -4.

**Examples**

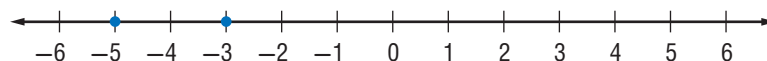
$-4 < -2$

$-2 > -4$

### EXAMPLE Compare Two Integers

- 1 Replace the ● with  $<$  or  $>$  to make  $-5 \bullet -3$  a true sentence.

Graph each integer on a number line.



Since -5 is to the left of -3,  $-5 < -3$ .

### ✓ CHECK Your Progress

Replace each ● with  $<$  or  $>$  to make a true sentence.

a.  $-8 \bullet -4$

b.  $5 \bullet -1$

c.  $-10 \bullet -13$



## TEST EXAMPLE

- 2 The elevations, in feet, for the lowest points in California, Oklahoma, Louisiana, and Kentucky are listed. Which list shows the elevation in order from highest to lowest?

State	Elevation (ft)
California	-282
Oklahoma	289
Louisiana	-8
Kentucky	257

- A 289, -282, 257, -8  
 B -8, 257, -282, 289  
 C -282, -8, 257, 289  
 D 289, 257, -8, -282

### Test-Taking Tip

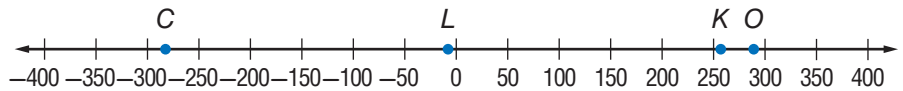
#### Eliminating Answer Choices

If you are unsure of the correct answer, eliminate the choices you know are incorrect. Then consider the remaining choices. You can eliminate choices B and C since those lists begin with a negative number.

### Read the Item

To order the integers, graph them on a number line.

### Solve the Item



Order the integers by reading from right to left: 289, 257, -8, -282. So, the answer is D.

### CHECK Your Progress

- d. A newspaper reporter lists the third round scores of the top five finishers in a golf tournament. Which list shows these scores from least to greatest?
- F 5, 2, 0, -1, -3                      H -3, -1, 0, 2, 5  
 G -1, -3, 0, 2, 5                        J 0, -1, 2, -3, 5

## CHECK Your Understanding

Example 1  
(p. 84)

Replace each ● with < or > to make a true sentence.

1.  $-4 \bullet -6$                       2.  $-2 \bullet 8$                       3.  $0 \bullet -10$

Example 2  
(p. 85)

Order the integers in each set from least to greatest.

4.  $\{-13, 9, -2, 0, 4\}$                       5.  $\{12, -16, -10, 19, -18\}$

6. **MULTIPLE CHOICE** The lowest temperatures in Hawaii, Illinois, Minnesota, and South Carolina are listed. Which list shows these temperatures in order from coldest to warmest?

- A -19, -36, -60, 12                      C -60, -36, -19, 12  
 B 12, -19, -36, -60                      D -60, -19, 12, -36





# Practice and Problem Solving

## HOMEWORK HELP

For Exercises	See Examples
7–14	1
15–20	2
35–36	2

Replace each ● with  $<$  or  $>$  to make a true sentence.

7.  $-7 \bullet -3$       8.  $-21 \bullet -12$       9.  $-6 \bullet -11$       10.  $-15 \bullet -33$   
 11.  $17 \bullet -20$       12.  $4 \bullet -4$       13.  $-5 \bullet 17$       14.  $-12 \bullet 8$

Order the integers in each set from least to greatest.

15.  $\{-8, 11, 6, -5, -3\}$       16.  $\{7, -2, 14, -9, 2\}$   
 17.  $\{5, -6, -7, -4, 1, 3\}$       18.  $\{-12, 15, 8, -15, -23, 10\}$

19. **ANALYZE TABLES** The ocean floor is divided into five zones according to how deep sunlight penetrates. Order the zones from closest to the surface to nearest to the ocean floor.

Zone	Beginning Ocean Depth
Abyssal	$-4,000$ m
Hadal	$-6,000$ m
Midnight	$-1,000$ m
Sunlight	$0$ m
Twilight	$-200$ m

20. **STOCK MARKET** Kevin's dad owns stock in five companies. The change in the stock value for each company was as follows: Company A,  $+12$ ; Company B,  $-5$ ; Company C,  $-25$ ; Company D,  $+18$ ; Company E,  $-10$ . Order the companies from the worst performing to best performing.

Replace each ● with  $<$ ,  $>$ , or  $=$  to make a true sentence.

21.  $-13 \bullet |-14|$       22.  $|36| \bullet -37$       23.  $-12 \bullet |12|$       24.  $|-29| \bullet |92|$

**FOOTBALL** For Exercises 25 and 26, use the information at the right. It shows the yardage gained each play for five plays.

Play	1	2	3	4	5
Yardage	10	$-2$	5	$-5$	20

25. Order the yardage from least to greatest.  
 26. Which run is the middle, or *median*, yardage?

27. **WEATHER** The wind chill index was invented in 1939 by Paul Siple, a polar explorer and authority on Antarctica. Wind chill causes the air to feel colder on human skin. Which feels colder: a temperature of  $10^\circ$  with a 15-mile-per-hour wind or a temperature of  $5^\circ$  with a 10 mile per hour wind?

Wind (mph)	WIND CHILL Temperature ( $^\circ$ F)				
	15	10	5	0	$-5$
5	7	1	$-5$	$-11$	$-16$
10	3	$-4$	$-10$	$-16$	$-22$
15	0	$-7$	$-13$	$-19$	$-26$
20	$-2$	$-9$	$-15$	$-23$	$-29$

Determine whether each sentence is *true* or *false*. If *false*, change one number to make the sentence true.

28.  $-8 > 5$       29.  $-7 < 0$       30.  $|5| < -6$       31.  $10 > |-8|$

## EXTRA PRACTICE

See pages 671, 705.

**H.O.T. Problems**

32. **NUMBER SENSE** If 0 is the greatest integer in a set of five integers, what can you conclude about the other four integers?
33. **CHALLENGE** What is the greatest integer value of  $n$  such that  $n < 0$ ?
34. **WRITING IN MATH** Develop a method for ordering a set of negative integers from least to greatest without the aid of a number line. Explain your method and use it to order the set  $\{-5, -8, -1, -3\}$ .


**TEST PRACTICE**

35. On a certain game show, contestants receive positive numbers of points for correct responses and negative numbers of points for incorrect responses. Which list gives the points a contestant received during one round of the game in order from highest to lowest?
- A  $-200, -400, -1000, 200, 600$   
 B  $600, -1000, -400, -200, 200$   
 C  $600, 200, -200, -400, -1000$   
 D  $-1000, -400, -200, 600, 200$

36. Which statement about the values shown is *not* true?

State	Low Temperature ( $^{\circ}\text{F}$ )
AR	$-29$
GA	$-17$
MS	$-19$
VA	$-30$
TX	$-23$

Source: *The World Almanac*

- F Virginia's record low is less than the record low for Arkansas.
- G Arkansas' record low is less than the record low for Georgia.
- H Mississippi's record low is greater than the record low for Texas.
- J Texas' record low is less than the record low for Arkansas.


**Spiral Review**

Write an integer for each situation. (Lesson 2-1)

37.  $9^{\circ}\text{C}$  below zero
38. a gain of 20 feet

**HOBBIES** For Exercises 39 and 40, use the following information. (Lesson 1-10)

Sophia estimates that she knits 6 rows of an afghan each hour.

39. Write an equation using two variables to represent the total number of rows  $r$  completed by Sophia after time  $t$ .
40. How many rows will Sophia have completed after 4 hours?


**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Graph each point on a vertical number line that goes from  $-10$  on the bottom to  $10$  at the top. (Lesson 2-1)

41.  $-3$                       42.  $0$                       43.  $4$                       44.  $-7$

# 2-3

## The Coordinate Plane

### MAIN IDEA

Graph points on a coordinate plane.

### New Vocabulary

coordinate plane  
quadrant  
x-axis  
y-axis  
origin  
ordered pair  
x-coordinate  
y-coordinate

### Math Online

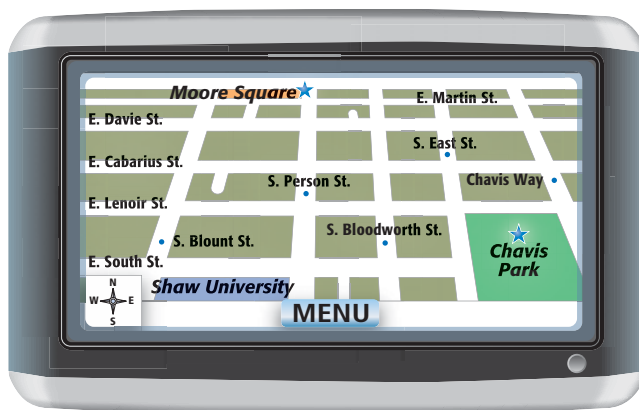
[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

### ▶ GET READY for the Lesson

**GPS** A GPS, or global positioning system, is a satellite based navigation system. A GPS map of Raleigh, North Carolina, is shown.

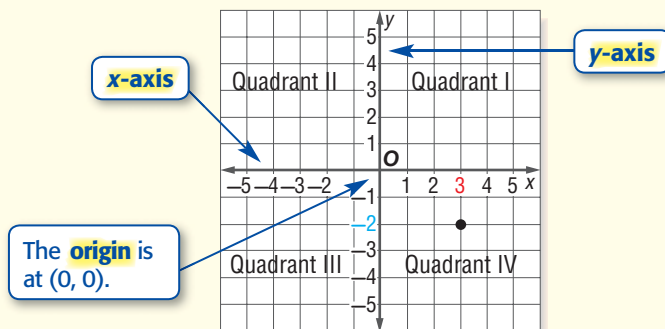
1. Suppose Mr. Diaz starts at Shaw University and drives 2 blocks north. Name the street he will cross.
2. Using the words *north*, *south*, *east*, and *west*, write directions to go from Chavis Park to Moore Square.



On a GPS, towns and streets are often located on a grid. In mathematics, we use a grid called a coordinate plane, to locate points. A **coordinate plane** is formed when two number lines intersect. The number lines separate the coordinate plane into four regions called **quadrants**.

### Coordinate Plane

### Key Concept



An **ordered pair** is a pair of numbers, such as  $(3, -2)$ , used to locate a point in the coordinate plane.

**x-coordinate** corresponds to a number on the x-axis.

$(3, -2)$

**y-coordinate** corresponds to a number on the y-axis.



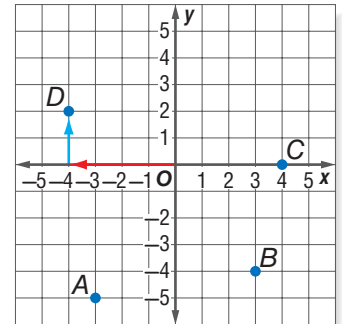
When locating an ordered pair, moving *right* or *up* on a coordinate plane is in the *positive* direction. Moving *left* or *down* is in the *negative* direction.

### EXAMPLE Naming Points Using Ordered Pairs

1 Write the ordered pair that corresponds to point  $D$ . Then state the quadrant in which the point is located.

- Start at the origin.
- Move left on the  $x$ -axis to find the  $x$ -coordinate of point  $D$ , which is  $-4$ .
- Move up to find the  $y$ -coordinate, which is  $2$ .

So, point  $D$  corresponds to the ordered pair  $(-4, 2)$ . Point  $D$  is located in Quadrant II.



### CHECK Your Progress

Write the ordered pair that corresponds to each point. Then state the quadrant or axis on which the point is located.

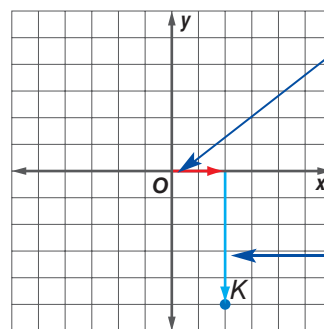
- a.  $A$                                       b.  $B$                                       c.  $C$

### EXAMPLE Graph an Ordered Pair

2 Graph and label point  $K$  at  $(2, -5)$ .

#### Reading Math

**Scale** When no numbers are shown on the  $x$ - or  $y$ -axis, you can assume that each square is 1 unit long on each side.



Start at the origin. The  $x$ -coordinate is  $2$ . So, move  $2$  units to the right.

Next, since the  $y$ -coordinate is  $-5$ , move  $5$  units down. Draw a dot and label  $K$ .

### CHECK Your Progress

On graph paper, draw a coordinate plane. Then graph and label each point.

- d.  $L(-4, 2)$                                       e.  $M(-5, -3)$                                       f.  $N(0, 1)$



## Real-World EXAMPLES

- 3 AQUARIUMS** A map can be divided into a coordinate plane where the  $x$ -coordinate represents how far to move right or left and the  $y$ -coordinate represents how far to move up or down. What exhibit is located at  $(6, 5)$ ?



### Real-World Link . . . .

New York Aquarium has a marine mammal exhibit that includes sea otters. Large male sea otters can grow to be over 4 feet in length.

Start at the origin. Move 6 units to the right and then 5 units up. Explore the Shore is located at  $(6, 5)$ .

- 4** In which quadrant is the Shark Exhibit located?

The Shark Exhibit is located in Quadrant IV.

### CHECK Your Progress

For Exercises g and h, use the map above.

- Find the ordered pair that represents the location of the Think Tank.
- What is located at the origin?

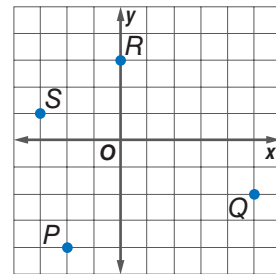
## CHECK Your Understanding

**Example 1** (p. 89) Write the ordered pair corresponding to each point graphed at the right. Then state the quadrant or axis on which each point is located.

- $P$
- $Q$
- $R$
- $S$

**Example 2** (p. 89) On graph paper, draw a coordinate plane. Then graph and label each point.

- $T(2, 3)$
- $U(-4, 6)$
- $V(-5, 0)$
- $W(1, -2)$



**Examples 3, 4**  
(p. 90)

**GEOGRAPHY** For Exercises 9 and 10, use the map in Example 3 above.

- What exhibit is located at  $(0, -3)$ ?
- In which quadrant is the Seaside Pavillion located?

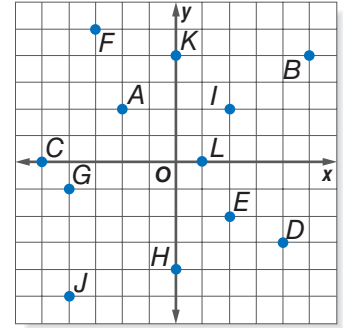


## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
11–22, 37	1
23–34, 38	2
35–36	3

Write the ordered pair corresponding to each point graphed at the right. Then state the quadrant or axis on which each point is located.

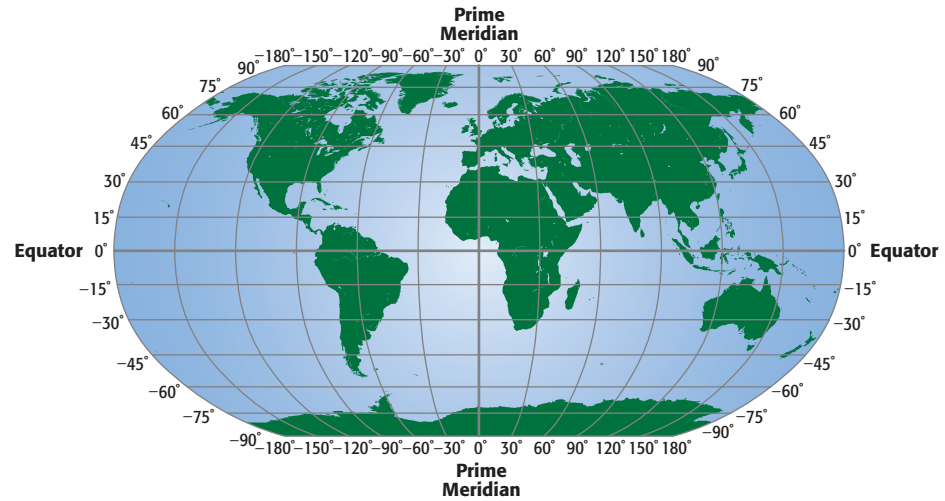


- |              |              |              |
|--------------|--------------|--------------|
| 11. <i>A</i> | 12. <i>B</i> | 13. <i>C</i> |
| 14. <i>D</i> | 15. <i>E</i> | 16. <i>F</i> |
| 17. <i>G</i> | 18. <i>H</i> | 19. <i>I</i> |
| 20. <i>J</i> | 21. <i>K</i> | 22. <i>L</i> |

On graph paper, draw a coordinate plane. Then graph and label each point.

- |                 |                 |                |                |
|-----------------|-----------------|----------------|----------------|
| 23. $M(5, 6)$   | 24. $N(-2, 10)$ | 25. $P(7, -8)$ | 26. $Q(3, 0)$  |
| 27. $R(-1, -7)$ | 28. $S(8, 1)$   | 29. $T(-3, 7)$ | 30. $U(5, -2)$ |
| 31. $V(0, 6)$   | 32. $W(-5, -7)$ | 33. $X(-4, 0)$ | 34. $Y(0, -5)$ |

**GEOGRAPHY** For Exercises 35–38, use the world map.



35. The world map can be divided into a coordinate plane where  $(x, y)$  represents (degrees longitude, degrees latitude). In what continent is the point  $(30^\circ$  longitude,  $-15^\circ$  latitude) located?
36. Which of the continents is located entirely in Quadrant II?
37. In what continent is the point  $(-90^\circ$  longitude,  $0^\circ$  latitude) located?
38. Name a continent on the map that is located entirely in Quadrant I.

On graph paper, draw a coordinate plane. Then graph and label each point.

- |                   |                                     |                          |
|-------------------|-------------------------------------|--------------------------|
| 39. $X(1.5, 3.5)$ | 40. $Y(3\frac{1}{4}, 2\frac{1}{2})$ | 41. $Z(2, 1\frac{2}{3})$ |
|-------------------|-------------------------------------|--------------------------|

42. **GEOMETRY** Graph four points on a coordinate plane so that they form a square when connected. Identify the ordered pairs.
43. **RESEARCH** Use the Internet or other resources to explain why the coordinate plane is sometimes called the Cartesian plane.



Determine whether each statement is *sometimes*, *always*, or *never* true. Explain or give a counterexample to support your answer.

**EXTRA PRACTICE**

See pages 672, 705.

- 44. Both  $x$ - and  $y$ -coordinates of a point in Quadrant III are negative.
- 45. The  $y$ -coordinate of a point that lies on the  $y$ -axis is negative.
- 46. The  $y$ -coordinate of a point in Quadrant II is negative.

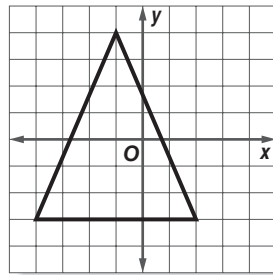
**H.O.T. Problems**

- 47. **OPEN ENDED** Create a display that shows how to determine in what quadrant a point is located without graphing. Then provide an example that demonstrates how your graphic is used.
- 48. **CHALLENGE** Find the possible locations for any ordered pair with  $x$ - and  $y$ -coordinates always having the same sign. Explain.
- 49. **WRITING IN MATH** Explain why the location of point  $A(1, -2)$  is different than the location of point  $B(-2, 1)$ .

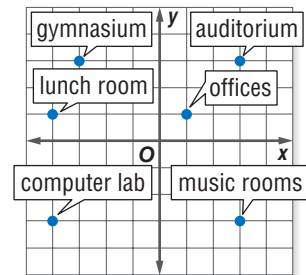


**TEST PRACTICE**

- 50. Which of the following points lie within the triangle graphed at the right?
  - A  $A(-4, -1)$
  - B  $B(1, 3)$
  - C  $C(-1, 2)$
  - D  $D(2, -2)$



- 51. What are the coordinates of the point that shows the location of the lunch room on the map?
  - F  $(4, -1)$
  - G  $(-4, 1)$
  - H  $(1, 4)$
  - J  $(1, -4)$



**Spiral Review**

Replace each  $\bullet$  with  $<$ ,  $>$ , or  $=$  to make a true sentence. (Lesson 2-2)

- 52.  $-8 \bullet -3$
- 53.  $26 \bullet -30$
- 54.  $14 \bullet |-15|$
- 55.  $-40 \bullet |40|$

- 56. Find the absolute value of  $-101$ . (Lesson 2-1)

- 57. **RUNNING** Salvador is training for a marathon. He runs 5 miles each day on weekdays and 8 miles each day on the weekends. How many miles does Salvador run in one week? (Lesson 1-1)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Add.

- 58.  $138 + 246$
- 59.  $814 + 512$
- 60.  $2,653 + 4,817$
- 61.  $6,003 + 5,734$

## Explore 2-4

# Algebra Lab Adding Integers

### MAIN IDEA

Use counters to model the addition of integers.

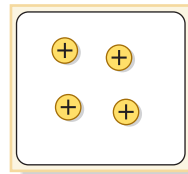
### Math Online

[glencoe.com](http://glencoe.com)

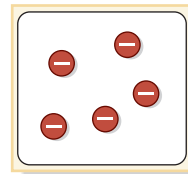
• Concepts In Motion

You can use positive and negative counters to model the addition of integers. The counter  $\oplus$  represents 1, and the counter  $\ominus$  represents  $-1$ . Remember that addition means *combining* two sets.

These counters represent  $+4$ .



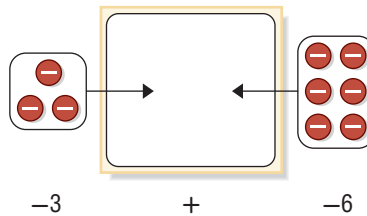
These counters represent  $-5$ .



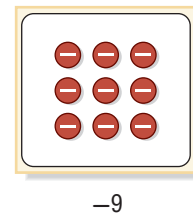
### ACTIVITY

1 Use counters to find  $-3 + (-6)$ .

Combine a set of 3 negative counters and a set of 6 negative counters.



Find the total number of counters.



So,  $-3 + (-6) = -9$ .

### CHECK Your Progress

Use counters or a drawing to find each sum.

a.  $5 + 6$

b.  $-3 + (-5)$

c.  $-5 + (-4)$

d.  $7 + 3$

e.  $-2 + (-5)$

f.  $-8 + (-6)$

The following two properties are important when modeling operations with integers.

- When one positive counter is paired with one negative counter, the result is called a **zero pair**. The value of a zero pair is 0.
- You can add or remove zero pairs from a mat because adding or removing zero does not change the value of the counters on the mat.



## ACTIVITIES

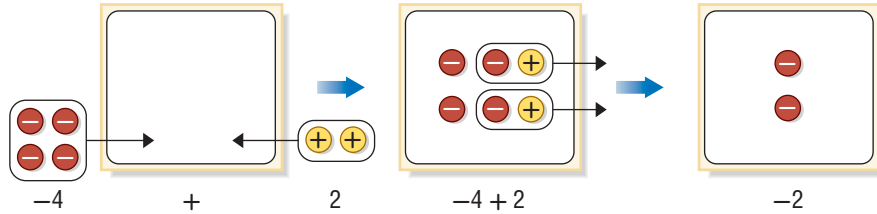
Use counters to find each sum.

2  $-4 + 2$

Combine 4 negatives with 2 positives.

Remove all zero pairs.

Find the remaining number of counters.



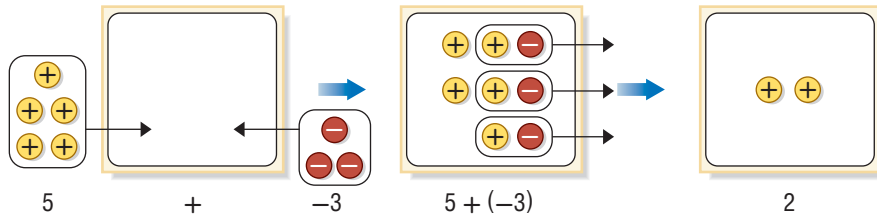
So,  $-4 + 2 = -2$ .

3  $5 + (-3)$

Combine 5 positives with 3 negatives.

Remove all zero pairs.

Find the remaining number of counters.



So,  $5 + (-3) = 2$ .

## ✓ CHECK Your Progress

Use counters or a drawing to find each sum.

g.  $-6 + 5$

h.  $3 + (-6)$

i.  $-2 + 7$

j.  $8 + (-3)$

k.  $-9 + 1$

l.  $-4 + 10$

## ANALYZE THE RESULTS

- Write two addition sentences where the sum is positive. In each sentence, one addend should be positive and the other negative.
- Write two addition sentences where the sum is negative. In each sentence, one addend should be positive and the other negative.
- MAKE A CONJECTURE** What is a rule you can use to determine how to find the sum of two integers with the same sign? two integers with different signs?

# 2-4

# Adding Integers

## MAIN IDEA

Add integers.

## New Vocabulary

**opposites**  
**additive inverse**

## Math Online

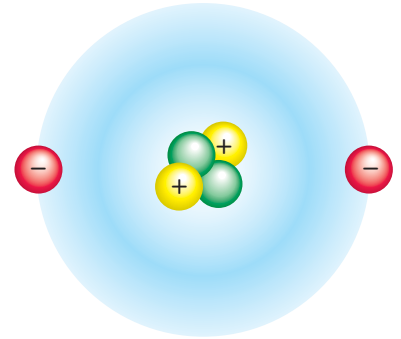
[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

## ▶ GET READY for the Lesson

**SCIENCE** Atoms are made of negative charges (electrons) and positive charges (protons). The helium atom shown has a total of 2 electrons and 2 protons.

1. Represent the electrons in an atom of helium with an integer.
2. Represent the protons in an atom of helium with an integer.
3. Each proton-electron pair has a value of 0. What is the total charge of an atom of helium?



Combining protons and electrons in an atom is similar to adding integers.

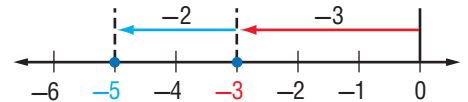
## EXAMPLE Add Integers with the Same Sign

**1** Find  $-3 + (-2)$ .

Use a number line.

- Start at 0.
- Move 3 units left to show  $-3$ .
- From there, move 2 units left to show  $-2$ .

So,  $-3 + (-2) = -5$ .



## ✓ CHECK Your Progress

a.  $-5 + (-7)$

b.  $-10 + (-4)$

These and other examples suggest the following rule.

## Add Integers with the Same Sign

### Key Concept

**Words** To add integers with the same sign, add their absolute values. The sum is:

- positive if both integers are positive.
- negative if both integers are negative.

**Examples**  $7 + 4 = 11$       $-7 + (-4) = -11$



## EXAMPLE Add Integers with the Same Sign

2 Find  $-26 + (-17)$ .

$$-26 + (-17) = -43 \quad \text{Both integers are negative, so the sum is negative.}$$

### CHECK Your Progress

c.  $-14 + (-16)$

d.  $23 + 38$

### Vocabulary Link . . . . . **Opposite**

**Everyday Use** something that is across from or is facing the other way, as in running the opposite way

**Math Use** two numbers that are the same distance from 0, but on opposite sides of 0 on the number line

••• The integers 5 and  $-5$  are called **opposites** because they are the same distance from 0, but on opposite sides of 0. Two integers that are opposites are also called **additive inverses**.

## Additive Inverse Property

Key Concept

**Words** The sum of any number and its additive inverse is 0.

**Examples**  $5 + (-5) = 0$

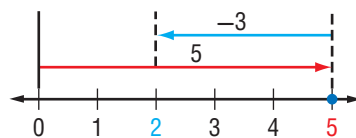
Number lines can also help you add integers with different signs.

## EXAMPLES Add Integers with Different Signs

3 Find  $5 + (-3)$ .

Use a number line.

- Start at zero.
- Move 5 units right.
- Then move 3 units left.

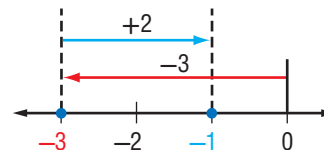


So,  $5 + (-3) = 2$ .

4 Find  $-3 + 2$ .

Use a number line.

- Start at zero.
- Move 3 units left.
- Then move 2 units right.



So,  $-3 + 2 = -1$ .

### CHECK Your Progress

e.  $6 + (-7)$

f.  $-15 + 19$

### Study Tip

**Look Back** You can review **absolute value** in Lesson 2-1.

## Add Integers with Different Signs

Key Concept

**Words** To add integers with different signs, subtract their absolute values. The sum is:

- positive if the positive integer's absolute value is greater.
- negative if the negative integer's absolute value is greater.

**Examples**  $9 + (-4) = 5$        $-9 + 4 = -5$



## EXAMPLES

## Add Integers with Different Signs

5 Find  $7 + (-1)$ .

$$7 + (-1) = 6$$

Subtract absolute values;  $7 - 1 = 6$ . Since 7 has the greater absolute value, the sum is positive.

6 Find  $-8 + 3$ .

$$-8 + 3 = -5$$

Subtract absolute values;  $8 - 3 = 5$ . Since  $-8$  has the greater absolute value, the sum is negative.

### Study Tip

**Properties** Using the Commutative, Associative, and Additive Inverse Properties allows the calculation to be as simple as possible.

7 Find  $2 + (-15) + (-2)$ .

$$\begin{aligned} 2 + (-15) + (-2) &= 2 + (-2) + (-15) && \text{Commutative Property (+)} \\ &= [2 + (-2)] + (-15) && \text{Associative Property (+)} \\ &= 0 + (-15) && \text{Additive Inverse Property} \\ &= -15 && \text{Additive Identity Property} \end{aligned}$$

### CHECK Your Progress

g.  $10 + (-12)$

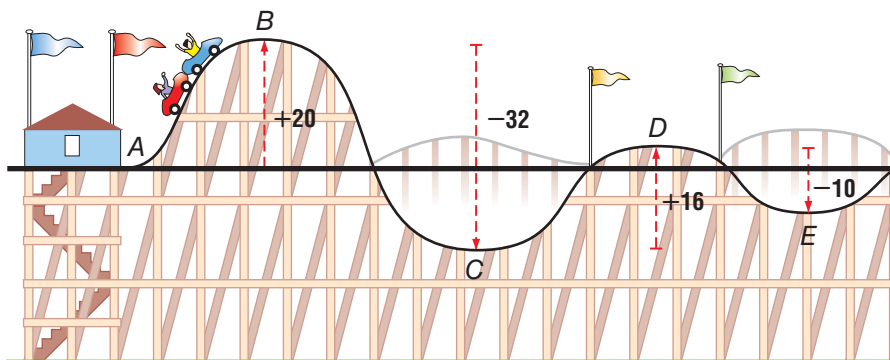
h.  $-13 + 18$

i.  $(-14) + (-6) + 6$



## Real-World EXAMPLE

8 **ROLLER COASTERS** The graphic shows the change in height at several points on a roller coaster. Write an addition sentence to find the height at point D in relation to point A.



$$\begin{aligned} 20 + (-32) + 16 &= 20 + 16 + (-32) && \text{Commutative Property (+)} \\ &= 36 + (-32) && 20 + 16 = 36 \\ &= 4 && \text{Subtract absolute values. Since 36 has the greater absolute value, the sum is positive.} \end{aligned}$$

The result is a positive integer. So, point D is 4 feet higher than point A.

### CHECK Your Progress

j. **WEATHER** The temperature is  $-3^\circ\text{F}$ . An hour later, it drops  $6^\circ$  and 2 hours later, it rises  $4^\circ$ . Write an addition sentence to describe this situation. Then find the sum and explain its meaning.



## CHECK Your Understanding

**Examples 1–6**  
(pp. 95–97)

**Add.**

1.  $-6 + (-8)$                       2.  $4 + 5$                                       3.  $-3 + 10$   
4.  $-15 + 8$                               5.  $7 + (-11)$                                   6.  $14 + (-6)$

**Example 7**  
(p. 97)

7.  $-17 + 20 + (-3)$                       8.  $15 + 9 + (-9)$

**Example 8**  
(p. 97)

9. **MONEY** Camilia owes her brother \$25, so she gives her brother the \$18 she earned dog-sitting for the neighbors. Write an addition sentence to describe this situation. Then find the sum and explain its meaning.

## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
10–13	1, 2
14–21	3–6
22–27	7
28–31	8

**Add.**

10.  $-22 + (-16)$     11.  $-10 + (-15)$     12.  $6 + 10$                       13.  $17 + 11$   
14.  $18 + (-5)$         15.  $13 + (-19)$     16.  $13 + (-7)$                       17.  $7 + (-20)$   
18.  $-19 + 24$         19.  $-12 + 10$         20.  $-30 + 16$                       21.  $-9 + 11$   
22.  $21 + (-21) + (-4)$     23.  $-8 + (-4) + 12$               24.  $-34 + 25 + (-25)$   
25.  $-16 + 16 + 22$         26.  $25 + 3 + (-25)$               27.  $7 + (-19) + (-7)$

**Write an addition expression to describe each situation. Then find each sum and explain its meaning.**

28. **SCUBA DIVING** Lena was scuba diving 14 meters below the surface of the water. She saw a nurse shark 3 meters above her.
29. **PELICANS** A pelican starts at 60 feet above sea level. It descends 60 feet to catch a fish.
30. **BANKING** Stephanie has \$152 in the bank. She withdraws \$20. Then she deposits \$84.
31. **FOOTBALL** A quarterback is sacked for a loss of 5 yards. On the next play, his team receives a penalty and loses 15 more yards. Then the team gains 12 yards on the third play.
32. **MONEY** Josephine is saving money for a new bike and has already saved \$17. Write the integers she should use to represent each entry.

Deposit	Withdrawal	Balance
\$15		■
	\$13	■
\$10		■
	\$4	■
\$13		■

**ALGEBRA** Evaluate each expression if  $x = -10$ ,  $y = 7$ , and  $z = -8$ .

33.  $x + 14$                       34.  $z + (-5)$   
35.  $x + y$                       36.  $x + z$

**EXTRA PRACTICE**

See pages 672, 705.

37. **FIND THE DATA** Refer to the Data File on pages 16–19 of your book. Choose some data and write a real-world problem in which you would add a positive and a negative integer. Then find the sum and explain its meaning.

**H.O.T. Problems**

38. **FIND THE ERROR** Beth and Jordan are finding  $-12 + 15$ . Who is correct? Explain your reasoning.



Beth

$$-12 + 15 = 3$$



Jordan

$$-12 + 15 = -3$$

**CHALLENGE** Simplify.

39.  $8 + (-8) + a$     40.  $x + (-5) + 1$     41.  $-9 + m + (-6)$     42.  $-1 + n + 7$

43. **WRITING IN MATH** Explain how you know whether a sum is positive, negative, or zero without actually adding.

**TEST PRACTICE**

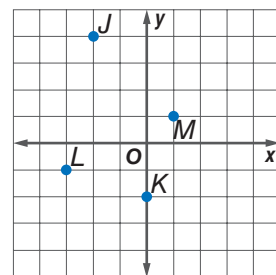
44. **SHORT RESPONSE** Find  $-8 + (-11)$ .
45. Find  $-8 + 7 + (-3)$ .  
 A  $-18$                       C  $2$   
 B  $-4$                          D  $18$
46. At 8 A.M., the temperature was  $3^{\circ}\text{F}$  below zero. By 1 P.M., the temperature rose  $14^{\circ}\text{F}$  and by 10 P.M. dropped  $12^{\circ}\text{F}$ . What was the temperature at 10 P.M.?  
 F  $5^{\circ}\text{F}$  above zero  
 G  $5^{\circ}\text{F}$  below zero  
 H  $1^{\circ}\text{F}$  above zero  
 J  $1^{\circ}\text{F}$  below zero

**Spiral Review**

Write the ordered pair for each point graphed at the right. Then name the quadrant or axis on which each point is located. (Lesson 2-3)

47. J                      48. K                      49. L                      50. M

51. Order 6,  $-3$ , 0, 4,  $-8$ , 1, and  $-4$  from least to greatest. (Lesson 2-2)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Subtract.

52.  $287 - 125$                       53.  $420 - 317$                       54.  $5,684 - 2,419$                       55.  $7,000 - 3,891$

# Mid-Chapter Quiz

Lessons 2-1 through 2-4

Write an integer for each situation. (Lesson 2-1)

- dropped 45 feet
  - a bank deposit of \$100
  - gained 8 pounds
  - lost a \$5 bill
5. **OCEANS** The deepest point in the world is the Mariana Trench in the Western Pacific Ocean at a depth of 35,840 feet below sea level. Write this depth as an integer. (Lesson 2-1)

Evaluate each expression. (Lesson 2-1)

- $|-16|$
- $|24|$
- $|-9| - |3|$
- $|-13| + |-1|$



10. **ANALYZE TABLES** The table shows the record low temperatures for January and February in Lincoln, Nebraska.

Month	Temperature (°F)
January	-33
February	-27

Source: University of Nebraska, Lincoln

Which month had the coldest temperature? (Lesson 2-2)



11. **MULTIPLE CHOICE** The local news records the following changes in average daily temperature for the past week:  $4^\circ$ ,  $-7^\circ$ ,  $-3^\circ$ ,  $2^\circ$ ,  $9^\circ$ ,  $-8^\circ$ ,  $1^\circ$ . Which list shows the temperatures from least to greatest? (Lesson 2-2)
- $9^\circ, 4^\circ, 2^\circ, 1^\circ, -3^\circ, -7^\circ, -8^\circ$
  - $-7^\circ, -8^\circ, 1^\circ, -3^\circ, 2^\circ, 4^\circ, 9^\circ$
  - $-8^\circ, -7^\circ, -3^\circ, 1^\circ, 2^\circ, 4^\circ, 9^\circ$
  - $-8^\circ, -7^\circ, 1^\circ, 2^\circ, 3^\circ, 4^\circ, 9^\circ$

Replace each  $\bullet$  with  $<$ ,  $>$ , or  $=$  to make a true sentence. (Lesson 2-2)

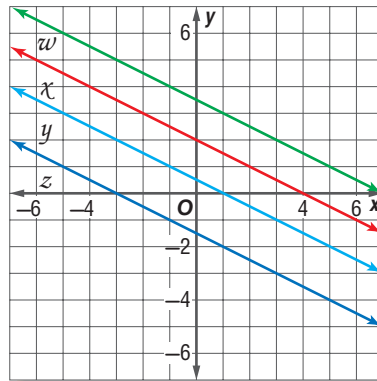
- $-4 \bullet 4$
- $-8 \bullet -11$
- $|-14| \bullet |3|$
- $|-12| \bullet |12|$

On graph paper, draw a coordinate plane. Then graph and label each point. (Lesson 2-3)

- $D(4, -3)$
- $E(-1, 2)$
- $F(0, -5)$
- $G(-3, 0)$



20. **MULTIPLE CHOICE** Which line contains the ordered pair  $(-1, 4)$ ? (Lesson 2-3)



- line  $w$
- line  $x$
- line  $y$
- line  $z$

Add. (Lesson 2-4)

- $3 + 4 + (-3)$
- $7 + (-11)$
- $-5 + (-6)$
- $8 + (-1) + 1$



25. **MULTIPLE CHOICE** Kendra deposited \$78 into her savings account. Two weeks later, she deposited a check for \$50 into her account and withdrew \$27. Which of the following expressions represents the amount of money left in her account? (Lesson 2-4)
- $\$78 + (-\$50) + (-\$27)$
  - $\$78 + (-\$50) + \$27$
  - $\$78 + \$50 + (-\$27)$
  - $\$78 + \$50 + \$27$

## Explore 2-5

# Algebra Lab Subtracting Integers

### MAIN IDEA

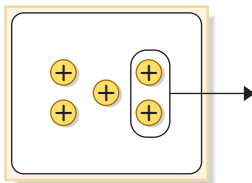
Use counters to model the subtraction of integers.

You can also use counters to model subtraction of integers. Remember one meaning of subtraction is to *take away*.

### ACTIVITY

Use counters to find each difference.

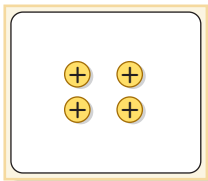
1  $5 - 2$



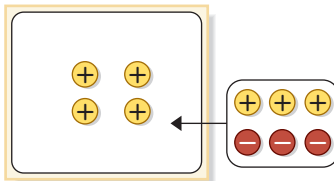
Place 5 positive counters on the mat. Remove 2 positive counters.

So,  $5 - 2 = 3$ .

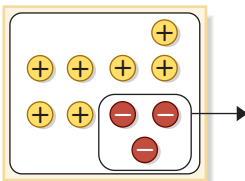
2  $4 - (-3)$



Place 4 positive counters on the mat. Remove 3 negative counters. However, there are 0 negative counters.



Add 3 zero pairs to the set.



Now you can remove 3 negative counters. Find the remaining number of counters.

So,  $4 - (-3) = 7$ .

### CHECK Your Progress

Use counters or a drawing to find each difference.

a.  $7 - 6$

b.  $5 - (-3)$

c.  $6 - (-3)$

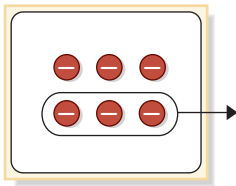
d.  $5 - 8$



## ACTIVITY

Use counters to find each difference.

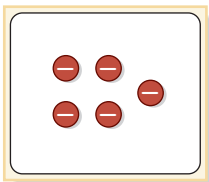
3  $-6 - (-3)$



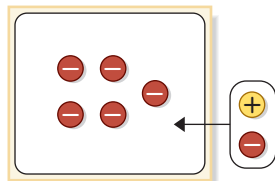
Place 6 negative counters on the mat. Remove 3 negative counters.

So,  $-6 - (-3) = -3$ .

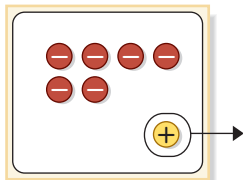
4  $-5 - 1$



Place 5 negative counters on the mat. Remove 1 positive counter. However, there are 0 positive counters.



Add 1 zero pair to the set.



Now you can remove 1 positive counter. Find the remaining number of counters.

So,  $-5 - 1 = -6$ .

## CHECK Your Progress

Use counters or a drawing to find each difference.

e.  $-6 - (-3)$

f.  $-7 - 3$

g.  $-5 - (-7)$

## Reading Math

### Minuends, Subtrahends, and Differences

In the subtraction sentence  $-5 - 1 = -6$ ,  $-5$  is the *minuend*,  $1$  is the *subtrahend*, and  $-6$  is the *difference*.

## ANALYZE THE RESULTS

- Write two subtraction sentences where the difference is positive. Use a combination of positive and negative integers.
- Write two subtraction sentences where the difference is negative. Use a combination of positive and negative integers.
- MAKE A CONJECTURE** Write a rule that will help you determine the sign of the difference of two integers.

# 2-5

# Subtracting Integers

### MAIN IDEA

Subtract integers.

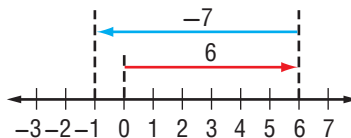
### Math Online

[glencoe.com](http://glencoe.com)

- Concepts In Motion
- Extra Examples
- Personal Tutor
- Self-Check Quiz

### MINI Lab

You can use a number line to model a subtraction problem.



1. Write a related addition sentence for the subtraction sentence.

Use a number line to find each difference. Write an equivalent addition sentence for each.

2.  $1 - 5$       3.  $-2 - 1$       4.  $-3 - 4$       5.  $0 - 5$

When you subtract 7, the result is the same as adding its opposite,  $-7$ .

$$6 - 7 = -1 \quad \text{opposite} \quad 6 + (-7) = -1$$

same result

This and other examples suggest the following rule.

### Subtract Integers

Key Concept

**Words** To subtract an integer, add its opposite.

**Examples**  $4 - 9 = 4 + (-9) = -5$        $7 - (-10) = 7 + (10) = 17$

### EXAMPLES Subtract Positive Integers

1 Find  $8 - 13$ .

$$\begin{aligned} 8 - 13 &= 8 + (-13) && \text{To subtract 13, add } -13. \\ &= -5 && \text{Simplify.} \end{aligned}$$

2 Find  $-10 - 7$ .

$$\begin{aligned} -10 - 7 &= -10 + (-7) && \text{To subtract 7, add } -7. \\ &= -17 && \text{Simplify.} \end{aligned}$$

### CHECK Your Progress

- a.  $6 - 12$       b.  $-20 - 15$       c.  $-22 - 26$

**EXAMPLES****Subtract Negative Integers****3** Find  $1 - (-2)$ .

$$\begin{aligned} 1 - (-2) &= 1 + 2 \\ &= 3 \end{aligned}$$

To subtract  $-2$ , add 2.  
Simplify.

**4** Find  $-10 - (-7)$ .

$$\begin{aligned} -10 - (-7) &= -10 + 7 \\ &= -3 \end{aligned}$$

To subtract  $-7$ , add 7.  
Simplify.

**CHECK Your Progress**

d.  $4 - (-12)$

e.  $-15 - (-5)$

f.  $18 - (-6)$

**EXAMPLE****Evaluate an Expression****5 ALGEBRA** Evaluate  $x - y$  if  $x = -6$  and  $y = -5$ .

$$\begin{aligned} x - y &= -6 - (-5) \\ &= -6 + (5) \\ &= -1 \end{aligned}$$

Replace  $x$  with  $-6$  and  $y$  with  $-5$ .  
To subtract  $-5$ , add 5.  
Simplify.

**CHECK Your Progress**Evaluate each expression if  $a = 5$ ,  $b = -8$ , and  $c = -9$ .

g.  $b - 10$

h.  $a - b$

i.  $c - a$

**Real-World EXAMPLE****6 SPACE** The temperatures on the Moon vary from  $-173^\circ\text{C}$  to  $127^\circ\text{C}$ . Find the difference between the maximum and minimum temperatures.

To find the difference in temperatures, subtract the lower temperature from the higher temperature.

**Estimate**  $100 + 200 = 300$ 

$$\begin{aligned} 127 - (-173) &= 127 + 173 \\ &= 300 \end{aligned}$$

To subtract  $-173$ , add 173.  
Simplify.

So, the difference between the temperatures is  $300^\circ\text{C}$ .**CHECK Your Progress**

j. **GEOGRAPHY** The Dead Sea's deepest part is 799 meters below sea level. A plateau to the east of the Dead Sea rises to about 1,340 meters above sea level. What is the difference between the top of the plateau and the deepest part of the Dead Sea?

**Real-World Link . . . . .**

The mean surface temperature on the Moon during the day is  $107^\circ\text{C}$ .

Source: Views of the Solar System

## CHECK Your Understanding

**Examples 1, 2**  
(p. 103)

**Subtract.**

1.  $14 - 17$   
3.  $-4 - 8$

2.  $10 - 30$   
4.  $-2 - 23$

**Examples 3, 4**  
(p. 104)

5.  $14 - (-10)$   
7.  $-3 - (-1)$

6.  $5 - (-16)$   
8.  $-11 - (-9)$

**Example 5**  
(p. 104)

**ALGEBRA** Evaluate each expression if  $p = 8$ ,  $q = -14$ , and  $r = -6$ .

9.  $r - 15$                       10.  $q - r$                       11.  $p - q$

**Example 6**  
(p. 104)

12. **EARTH SCIENCE** The sea-surface temperatures range from  $-2^{\circ}\text{C}$  to  $31^{\circ}\text{C}$ . Find the difference between the maximum and minimum temperatures.

## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
13-16, 21-24	1, 2
17-20, 25-28	3, 4
29-36	5
37-40	6

**Subtract.**

- |                  |                  |                   |                   |
|------------------|------------------|-------------------|-------------------|
| 13. $0 - 10$     | 14. $13 - 17$    | 15. $-9 - 5$      | 16. $-8 - 9$      |
| 17. $4 - (-19)$  | 18. $27 - (-8)$  | 19. $-11 - (-42)$ | 20. $-27 - (-19)$ |
| 21. $12 - 26$    | 22. $31 - 48$    | 23. $-25 - 5$     | 24. $-44 - 41$    |
| 25. $52 - (-52)$ | 26. $15 - (-14)$ | 27. $-27 - (-33)$ | 28. $-18 - (-20)$ |

**ALGEBRA** Evaluate each expression if  $f = -6$ ,  $g = 7$ , and  $h = 9$ .

- |             |             |                 |                |
|-------------|-------------|-----------------|----------------|
| 29. $g - 7$ | 30. $f - 6$ | 31. $-h - (-9)$ | 32. $f - g$    |
| 33. $h - f$ | 34. $g - h$ | 35. $5 - f$     | 36. $4 - (-g)$ |

**ANALYZE TABLES** For Exercises 37-40, use the information below.

State	California	Georgia	Louisiana	New Mexico	Texas
Lowest Elevation (ft)	-282	0	-8	2,842	0
Highest Elevation (ft)	14,494	4,784	535	13,161	8,749

37. What is the difference between the highest elevation in Texas and the lowest elevation in Louisiana?
38. Find the difference between the lowest elevation in New Mexico and the lowest elevation in California.
39. Find the difference between the highest elevation in Georgia and the lowest elevation in California.
40. What is the difference between the lowest elevations in Texas and Louisiana?

### EXTRA PRACTICE

See pages 672, 705.

**ALGEBRA** Evaluate each expression if  $h = -12$ ,  $j = 4$ , and  $k = 15$ .

41.  $-j + h - k$                       42.  $|h - j|$                       43.  $k - j - h$

**H.O.T. Problems**

44. **OPEN ENDED** Write a subtraction sentence using integers. Then, write the equivalent addition sentence, and explain how to find the sum.
45. **FIND THE ERROR** Alicia and Mei are finding  $-15 - (-18)$ . Who is correct? Explain your reasoning.

Alicia:  $-15 - (-18) = -15 + (-18) = -33$

Mei:  $-15 - (-18) = -15 + (18) = 3$

46. **CHALLENGE** True or False? When  $n$  is a negative integer,  $n - n = 0$ .
47. **WRITING IN MATH** Explain how additive inverses are used in subtraction.

**TEST PRACTICE**

48. Which sentence about integers is *not* always true?
- A positive  $-$  positive = positive  
B positive  $+$  positive = positive  
C negative  $+$  negative = negative  
D positive  $-$  negative = positive
49. Morgan drove from Los Angeles (elevation 330 feet) to Death Valley (elevation  $-282$  feet). What is the difference in elevation between Los Angeles and Death Valley?
- F 48 feet      H 582 feet  
G 148 feet      J 612 feet

**Spiral Review**

Add. (Lesson 2-4)

50.  $10 + (-3)$       51.  $-2 + (-9)$       52.  $-7 + (-6)$       53.  $-18 + 4$

54. In which quadrant does the ordered pair  $(5, -6)$  lie? (Lesson 2-3)

55. **NUMBERS** A number times 2 is added to 7. The result is 23. What is the number? Use the *guess and check* strategy. (Lesson 1-5)

**GET READY for the Next Lesson**

Add. (Lesson 2-4)

56.  $-6 + (-6) + (-6) + (-6)$       57.  $-11 + (-11) + (-11)$   
58.  $-2 + (-2) + (-2) + (-2)$       59.  $-8 + (-8) + (-8)$

# 2-6

# Multiplying Integers

## MAIN IDEA

Multiply integers.

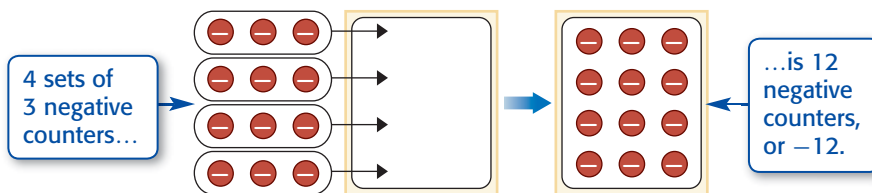
## Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

## MINI Lab

Counters can be used to multiply integers.



1. Write a multiplication sentence that describes the model above.

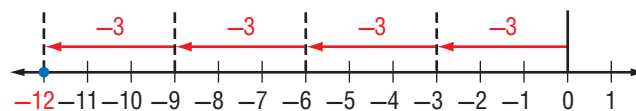
Find each product using counters or a drawing.

2.  $3(-2)$       3.  $4(-3)$       4.  $1(-7)$       5.  $5(-2)$

Remember that multiplication is the same as repeated addition.

$$4(-3) = (-3) + (-3) + (-3) + (-3) \quad -3 \text{ is used as an addend four times.}$$

$$= -12$$



By the Commutative Property of Multiplication,  $4(-3) = -3(4)$ .

## Multiply Integers with Different Signs

Key Concept

**Words** The product of two integers with different signs is negative.

**Examples**  $6(-4) = -24$        $-5(7) = -35$

## EXAMPLES

## Multiply Integers with Different Signs

1 Find  $3(-5)$ .

$3(-5) = -15$  The integers have different signs. The product is negative.

2 Find  $-6(8)$ .

$-6(8) = -48$  The integers have different signs. The product is negative.

## CHECK Your Progress

a.  $9(-2)$

b.  $-7(4)$



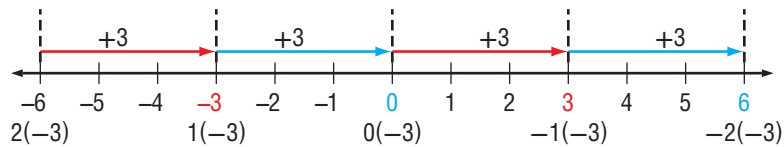
The product of two positive integers is positive. You can use a pattern to find the sign of the product of two negative integers.

### Study Tip

**Multiplying by Zero**  
The Multiplicative Property of Zero states that when any number is multiplied by zero, the product is zero.

positive $\times$ negative = negative	$(2)(-3) = -6$	} +3
	$(1)(-3) = -3$	
	$(0)(-3) = 0$	
negative $\times$ negative = positive	$(-1)(-3) = 3$	} +3
	$(-2)(-3) = 6$	

Each product is 3 more than the previous product. This pattern can also be shown on a number line.



These and other examples suggest the following rule.

## Multiply Integers with Same Sign

Key Concept

**Words** The product of two integers with the same sign is positive.

**Examples**  $2(6) = 12$       $-10(-6) = 60$

### EXAMPLES

### Multiply Integers with the Same Sign

**3** Find  $-11(-9)$ .

$$-11(-9) = 99 \quad \text{The integers have the same sign. The product is positive.}$$

**4** Find  $(-4)^2$ .

$$\begin{aligned} (-4)^2 &= (-4)(-4) && \text{There are two factors of } -4. \\ &= 16 && \text{The product is positive.} \end{aligned}$$

**5** Find  $-3(-4)(-2)$ .

$$\begin{aligned} -3(-4)(-2) &= [-3(-4)](-2) && \text{Associative Property} \\ &= 12(-2) && -3(-4) = 12 \\ &= -24 && 12(-2) = -24 \end{aligned}$$

### CHECK Your Progress

c.  $-12(-4)$

d.  $(-5)^2$

e.  $-7(-5)(-3)$

### Study Tip

**Look Back**  
You can review exponents in Lesson 1-2.



### Real-World Link . . . .

The MIR submersible that explored the Titanic shipwreck was able to descend to  $-20,000$  feet.

Source: Space Adventures



## Real-World EXAMPLE

- 6 SUBMERSIBLES** A submersible is diving from the surface of the water at a rate of 90 feet per minute. What is the depth of the submersible after 7 minutes?

If the submersible descends 90 feet per minute, then after 7 minutes, the vessel will be at  $7(-90)$  or  $-630$  feet. Thus, the submersible will descend to 630 feet below the surface.



### CHECK Your Progress

- f. **MONEY** Mr. Simon's bank automatically deducts a \$4 monthly maintenance fee from his savings account. What integer represents a change in his savings account from one year of fees?

Negative numbers are often used when evaluating algebraic expressions.

## EXAMPLE

## Evaluate Expressions

- 7 ALGEBRA** Evaluate  $pqr$  if  $p = -3$ ,  $q = 4$ , and  $r = -1$ .

$$\begin{aligned} pqr &= -3(4)(-1) && \text{Replace } p \text{ with } -3, q \text{ with } 4, \text{ and } r \text{ with } -1. \\ &= (-12)(-1) && \text{Multiply } -3 \text{ and } 4. \\ &= 12 && \text{Multiply } -12 \text{ and } -1. \end{aligned}$$



### CHECK Your Progress

- g. Evaluate  $xyz$  if  $x = -7$ ,  $y = -4$ , and  $z = 2$ .



## CHECK Your Understanding

**Examples 1, 2**  
(p. 107)

**Multiply.**

1.  $6(-10)$

2.  $11(-4)$

3.  $-2(14)$

4.  $-8(5)$

**Examples 3–5**  
(p. 108)

**Multiply.**

5.  $-15(-3)$

6.  $-7(-9)$

7.  $(-8)^2$

8.  $(-3)^3$

9.  $-1(-3)(-4)$

10.  $2(4)(5)$

**Example 6**  
(p. 109)

11. **MONEY** Tamera owns 100 shares of a certain stock. Suppose the price of the stock drops by \$3 per share. Write a multiplication expression to find the change in Tamera's investment. Explain the answer.

**Example 7**  
(p. 109)

**ALGEBRA** Evaluate each expression if  $f = -1$ ,  $g = 7$ , and  $h = -10$ .

12.  $5f$

13.  $fgh$





# Practice and Problem Solving

## HOMESCHOOL HELP

For Exercises	See Examples
14–19, 28	1, 2
20–27, 29	3–5
30–37	7
38–39	6

Multiply.

14.  $8(-12)$       15.  $11(-20)$       16.  $-15(4)$       17.  $-7(10)$   
 18.  $-7(11)$       19.  $25(-2)$       20.  $-20(-8)$       21.  $-16(-5)$   
 22.  $(-6)^2$       23.  $(-5)^3$       24.  $(-4)^3$       25.  $(-9)^2$   
 26.  $-4(-2)(-8)$       27.  $-9(-1)(-5)$   
 28. Find the product of 10 and  $-10$ .      29. Find  $-7$  squared.

**ALGEBRA** Evaluate each expression if  $w = 4$ ,  $x = -8$ ,  $y = 5$ , and  $z = -3$ .

30.  $-4w$       31.  $3x$       32.  $xy$       33.  $xz$   
 34.  $7wz$       35.  $-2wx$       36.  $xyz$       37.  $wyx$

Write a multiplication expression to represent each situation. Then find each product and explain its meaning.

38. **ECOLOGY** Wave erosion causes a certain coastline to recede at a rate of 3 centimeters each year. This occurs uninterrupted for a period of 8 years.  
 39. **EXERCISE** Ethan burns 650 Calories when he runs for 1 hour. Suppose he runs 5 hours in one week.

**ALGEBRA** Evaluate each expression if  $a = -6$ ,  $b = -4$ ,  $c = 3$ , and  $d = 9$ .

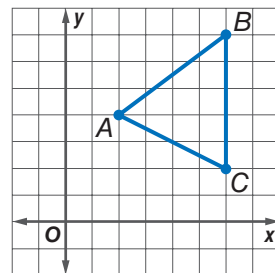
40.  $-3a^2$       41.  $-cd^2$       42.  $-2a + b$       43.  $b^2 - 4ac$

44. **BANKING** Tamika's aunt writes a check for \$150 each month for her car loan. She writes another check for \$300 twice a year to pay for car insurance. Write an expression involving multiplication and addition to describe how these expenses affect her checking account balance on a yearly basis. Then evaluate the expression and explain its meaning.

45. **FIND THE DATA** Refer to the Data File on pages 16–19 of your book. Choose some data and write a real-world problem in which you would multiply integers.

**GEOMETRY** For Exercises 46–48, use the graph at the right.

46. Name the ordered pairs for  $A$ ,  $B$ , and  $C$ . Multiply each  $x$ - and  $y$ -coordinate by  $-1$  to get three new ordered pairs.  
 47. Graph the ordered pairs and connect them to form a new triangle. Describe its position with respect to the original triangle.



48. In which quadrant would a new triangle lie if only the  $y$ -coordinates of the original triangle are multiplied by  $-1$ ?

## EXTRA PRACTICE

See pages 673, 705.

### H.O.T. Problems

49. **OPEN ENDED** Write a multiplication sentence with a product of  $-18$ .
50. **NUMBER SENSE** Explain how to evaluate each expression as simply as possible.
- a.  $(-9)(-6)(15)(-7 + 7)$       b.  $(-15)(-26) + (-15)(25)$
51. **CHALLENGE** Evaluate  $(-1)^{50}$ . Explain your reasoning.
52. **SELECT A TECHNIQUE** Luis is trying to determine whether the product of three negative integers is negative or positive. Which of the following techniques might he use to determine the answer? Justify your selections. Then provide an example that illustrates the answer.
- mental math      number sense      estimation
53. **WRITING IN MATH** Explain when the product of three integers is positive.

### TEST PRACTICE

54. The temperature drops 2 degrees per hour for 3 hours. Which expression does *not* describe the change in temperature?
- A  $-2(3)$       C  $-2 - 2 - 2$   
B  $-2 + (-2) + (-2)$       D  $2(3)$
55. Which of the following numbers is the 7th number in the sequence shown?
- $1, -2, 4, -8, 16, \dots$
- F  $-64$       H  $32$   
G  $-32$       J  $64$

### Spiral Review

56. **TEMPERATURE** The highest and lowest recorded temperatures in Europe are  $122^{\circ}\text{F}$  and  $-67^{\circ}\text{F}$ . Find the difference in these temperatures. (Lesson 2-5)

**Subtract.** (Lesson 2-5)

57.  $-25 - (-33)$       58.  $-6 - 14$       59.  $9 - 30$       60.  $13 - (-12)$

**ALGEBRA** Evaluate each expression if  $x = -4$ ,  $y = 6$ , and  $z = 1$ . (Lesson 2-4)

61.  $x + (-2)$       62.  $-1 + z$       63.  $-15 + y$       64.  $x + y$

65. **PENGUINS** The Emperor penguin's average height is 51 inches, and the Adelie's average height is 18 inches. Write and solve an addition equation to find how much taller Emperor penguins are than Adelie penguins. (Lesson 1-7)

### ▶ GET READY for the Next Lesson

66. **NUMBERS** A number is multiplied by  $-4$ . Then 15 is added to the product, and the result is 3. What is the number? Use the *guess and check* strategy. (Lesson 1-5)

# 2-7

## Problem-Solving Investigation

**MAIN IDEA:** Solve problems by looking for a pattern.

### P.S.I. TEAM +

#### e-Mail: LOOK FOR A PATTERN

**LAURA:** I've been practicing free throws every day after school to get ready for basketball tryouts. Now, I can make 3 free throws out of every 5 attempts I make.

**YOUR MISSION:** Look for a pattern to find the number of free throws Laura can make after 30 attempts.



<b>Understand</b>	Laura can make an average of 3 free throws out of every 5 attempts. You need to find the number of free throws she can make after 30 attempts.																												
<b>Plan</b>	Look for a pattern. Then extend the pattern to find the solution.																												
<b>Solve</b>	<p>Laura can make 3 free throws out of every 5 she attempts. Extend the pattern.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">+3</td> <td style="text-align: center;">+3</td> <td style="text-align: center;">+3</td> <td style="text-align: center;">+3</td> <td style="text-align: center;">+3</td> <td></td> </tr> <tr> <td style="background-color: #c6e0b4;">Free throws</td> <td>3</td> <td>6</td> <td>9</td> <td>12</td> <td>15</td> <td>18</td> </tr> <tr> <td style="background-color: #c6e0b4;">Attempts</td> <td>5</td> <td>10</td> <td>15</td> <td>20</td> <td>25</td> <td>30</td> </tr> <tr> <td></td> <td style="text-align: center;">+5</td> <td style="text-align: center;">+5</td> <td style="text-align: center;">+5</td> <td style="text-align: center;">+5</td> <td style="text-align: center;">+5</td> <td></td> </tr> </table> <p>She can make 18 free throws out of 30 attempts.</p>		+3	+3	+3	+3	+3		Free throws	3	6	9	12	15	18	Attempts	5	10	15	20	25	30		+5	+5	+5	+5	+5	
	+3	+3	+3	+3	+3																								
Free throws	3	6	9	12	15	18																							
Attempts	5	10	15	20	25	30																							
	+5	+5	+5	+5	+5																								
<b>Check</b>	She makes free throws a little more than half the time. Since 18 is a little more than 15, the answer is reasonable. ✓																												

### Analyze The Strategy

1. Explain when you would use the *look for a pattern* strategy to solve a problem.
2. Describe how to solve a problem using the *look for a pattern* strategy.
3. **WRITING IN MATH** Write a problem that could be solved by looking for a pattern.

## Mixed Problem Solving

**EXTRA PRACTICE**

See pages 673, 705.

Use the *look for a pattern* strategy to solve Exercises 4–6.

4. **DISPLAYS** A display of cereal boxes is stacked as shown below.



If the display contains 7 rows of boxes and the top three rows are shown, how many boxes are in the display?

5. **MONEY** Peter is saving money to buy an MP3 player. After one month, he has \$50. After 2 months, he has \$85. After 3 months, he has \$120. After 4 months, he has \$155. He plans to keep saving at the same rate. How long will it take Peter to save enough money to buy an MP3 player that costs \$295?
6. **INSECTS** The table shows how many times a cricket chirps at different temperatures. About how many times will a cricket chirp when the temperature is  $60^{\circ}\text{F}$ ?

Outside Temperature ( $^{\circ}\text{F}$ )	Chirps per Minute
$85^{\circ}\text{F}$	180
$80^{\circ}\text{F}$	160
$75^{\circ}\text{F}$	140
$70^{\circ}\text{F}$	120



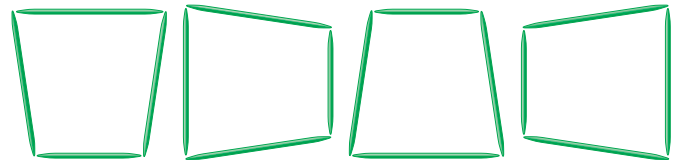
Use any strategy to solve Exercises 7–14. Some strategies are shown below.

### PROBLEM-SOLVING STRATEGIES

- Guess and check
- Look for a pattern

7. **COINS** Adelina has six coins that total \$0.86. What are the coins?

8. **ELEVATION** The lowest point in Mexico is Laguna Salada with an elevation of  $-10$  meters. The highest point in Mexico is Volcan Pico de Orizaba with an elevation of 5,700 meters. What is the difference in these elevations?
9. **MONEY** While on vacation, Edmundo sent postcards and letters to his friends. He spent \$3.42 on postage. A stamp for a letter costs 41¢, and a stamp for a postcard costs 26¢. How many postcards and letters did he send?
10. **GEOMETRY** What is the next figure in the pattern shown?



11. **POPULATION** The total land area of North Carolina is about 48,711 square miles. If an average of 183 persons were living in each square mile of North Carolina in 2007, what was the population of North Carolina in 2007?
12. **GOLF** Allie's golf scores for the first five holes are given in the table. What is her total score after the first five holes?

Hole	Score
1	0
2	1
3	-1
4	-2
5	3

13. **FLOWERS** A sunflower grows to be about 252 centimeters tall in 3 months. What is the average rate of growth per month?
14. **NUMBERS** Determine the next three numbers in the pattern below.  
48, 42, 36, 30, 24, ...

# 2-8

## Dividing Integers

### MAIN IDEA

Divide integers.

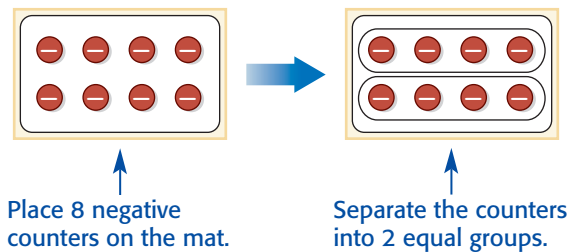
### Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

### MINI Lab

You can use counters to model division of integers. Follow these steps to find  $-8 \div 2$ .



There are 4 negative counters in each group. So,  $-8 \div 2 = -4$ .

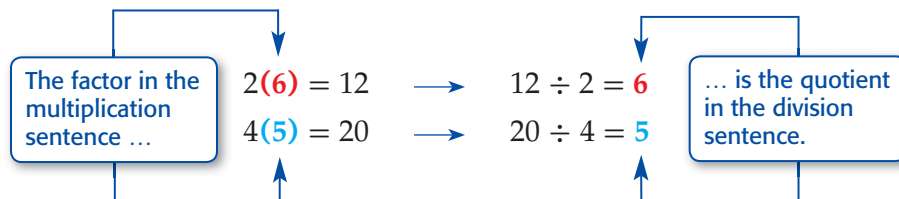
Find each quotient using counters or a drawing.

1.  $-6 \div 2$

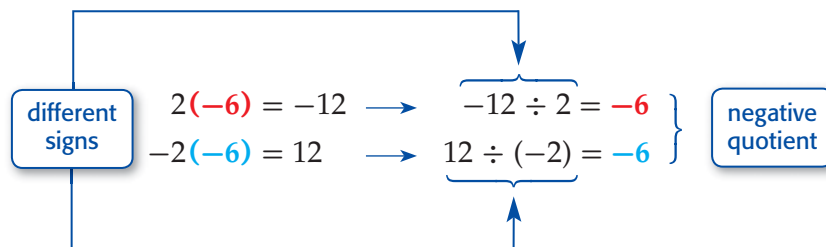
2.  $-12 \div 3$



Division of numbers is related to multiplication. When finding the quotient of two integers, you can use a related multiplication sentence.



Since multiplication and division sentences are related, you can use them to find the quotient of integers with different signs.



These related sentences lead to the following rule.

### Dividing Integers with Different Signs

Key Concept

**Words** The quotient of two integers with different signs is negative.

**Examples**  $33 \div (-11) = -3$      $-64 \div 8 = -8$



## EXAMPLES

## Dividing Integers with Different Signs

1 Find  $80 \div (-10)$ . The integers have different signs.

$$80 \div (-10) = -8 \quad \text{The quotient is negative.}$$

2 Find  $\frac{-55}{11}$ . The integers have different signs.

$$\frac{-55}{11} = -5 \quad \text{The quotient is negative.}$$

## CHECK Your Progress

a.  $20 \div (-4)$

b.  $\frac{-81}{9}$

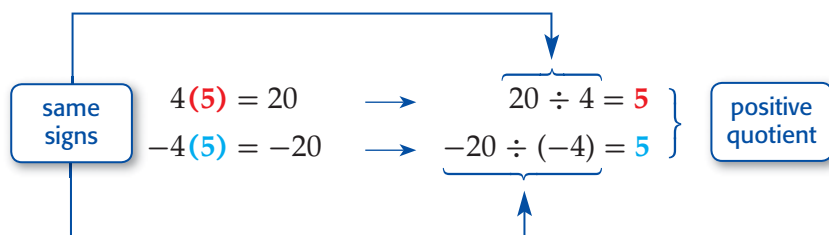
c.  $-45 \div 9$

### Study Tip

#### Dividing Integers

Dividing integers with same or different signs follow the same rules as the ones for multiplication.

You can also use multiplication and division sentences to find the quotient of integers with the same sign.



These related sentences lead to the following rule.

## Divide Integers with the Same Sign

### Key Concept

**Words** The quotient of two integers with the same sign is positive.

**Examples**  $15 \div 5 = 3$        $-64 \div (-8) = 8$

## EXAMPLES

## Dividing Integers with the Same Sign

3 Find  $-14 \div (-7)$ . The integers have the same sign.

$$-14 \div (-7) = 2 \quad \text{The quotient is positive.}$$

4 **ALGEBRA** Evaluate  $-16 \div x$  if  $x = -4$ .

$$\begin{aligned} -16 \div x &= -16 \div (-4) && \text{Replace } x \text{ with } -4. \\ &= 4 && \text{Divide. The quotient is positive.} \end{aligned}$$

## CHECK Your Progress

d.  $-24 \div (-4)$

e.  $-9 \div (-3)$

f.  $\frac{-28}{-7}$

g. **ALGEBRA** Evaluate  $a \div b$  if  $a = -9$  and  $b = -3$ .



### Real-World Link . . . . .

An adult koala is 25–30 inches long and weighs 15–30 pounds.

Source: Koala Express

## Real-World EXAMPLE

**5 ANIMALS** Ten years ago, the estimated Australian koala population was 1,000,000. Today there are about 100,000 koalas. Use the expression  $\frac{N - P}{10}$ , where  $N$  represents the new population and  $P$  the previous population to find the average change in the koala population per year for the 10-year period.

$$\begin{aligned} \frac{N - P}{10} &= \frac{100,000 - 1,000,000}{10} && \text{Replace } N \text{ with } 100,000 \text{ and } P \text{ with } 1,000,000. \\ &= \frac{-900,000}{10} \text{ or } -90,000 && \text{Divide.} \end{aligned}$$

The koala population has changed by  $-90,000$  per year.

### ✓ CHECK Your Progress

**h. WEATHER** The average temperature in January for North Pole, Alaska, is  $-24.4^\circ\text{C}$ . Use the expression  $\frac{9C + 160}{5}$ , where  $C$  represents the number of degrees Celsius, to find this temperature in degrees Fahrenheit.

## Operations with Integers

### Concept Summary

Operation	Rule
<b>Add</b>	<b>Same Sign:</b> Add absolute values. The sum has the same sign as the integers. <b>Different Signs:</b> Subtract absolute values. The sum has the sign of the integer with greater absolute value.
<b>Subtract</b>	To subtract an integer, add its opposite.
<b>Multiply and Divide</b>	<b>Same Signs:</b> The product or quotient is positive. <b>Different Signs:</b> The product or quotient is negative.

### ✓ CHECK Your Understanding

**Examples 1–3**  
(p. 115)

Divide.

1.  $32 \div (-8)$

2.  $-16 \div 2$

3.  $\frac{42}{-7}$

4.  $-30 \div (-5)$

5.  $55 \div 11$

6.  $\frac{-16}{-4}$

**Example 4**  
(p. 115)

**ALGEBRA** Evaluate each expression if  $x = 8$  and  $y = -5$ .

7.  $15 \div y$

8.  $xy \div (-10)$



**Example 5**  
(p. 116)

**9. TEMPERATURE** The lowest recorded temperature in Wisconsin is  $-55^\circ\text{F}$  on February 4, 1996. Use the expression  $\frac{5(F - 32)}{9}$  to find this temperature in degrees Celsius. Round to the nearest tenth.



## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
10–13, 16–19	1, 2
14–15, 20–23	3
24–31	4
32–33	5

Divide.

10.  $50 \div (-5)$       11.  $56 \div (-8)$       12.  $-18 \div 9$       13.  $-36 \div 4$
14.  $-15 \div (-3)$       15.  $-100 \div (-10)$       16.  $\frac{22}{-2}$       17.  $\frac{84}{-12}$
18.  $\frac{-26}{13}$       19.  $\frac{-27}{3}$       20.  $\frac{-21}{-7}$       21.  $\frac{-54}{-6}$
22. Divide  $-200$  by  $-100$ .      23. Find the quotient of  $-65$  and  $-13$ .

**ALGEBRA** Evaluate each expression if  $r = 12$ ,  $s = -4$ , and  $t = -6$ .

24.  $-12 \div r$       25.  $72 \div t$       26.  $r \div s$       27.  $rs \div 16$
28.  $\frac{t-r}{3}$       29.  $\frac{8-r}{-2}$       30.  $\frac{s+t}{5}$       31.  $\frac{t+9}{-3}$

32. **MONEY** Last year, Mr. Engle's total income was \$52,000, while his total expenses were \$53,800. Use the expression  $\frac{I-E}{12}$ , where  $I$  represents total income and  $E$  represents total expenses, to find the average difference between his income and expenses each month.

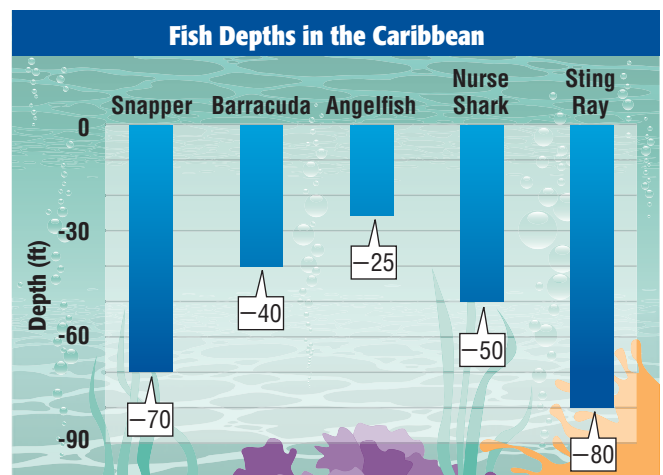
33. **SCIENCE** The boiling point of water is affected by changes in elevation. Use the expression  $\frac{-2A}{1,000}$ , where  $A$  represents the altitude in feet, to find the number of degrees Fahrenheit the boiling point of water changes at an altitude of 5,000 feet.

**ALGEBRA** Evaluate each expression if  $d = -9$ ,  $f = 36$ , and  $g = -6$ .

34.  $\frac{-f}{d}$       35.  $\frac{12 - (-f)}{-g}$       36.  $\frac{f^2}{d^2}$       37.  $g^2 \div f$

38. **PLANETS** The temperature on Mars ranges widely from  $-207^\circ\text{F}$  at the winter pole to almost  $80^\circ\text{F}$  on the dayside during the summer. Use the expression  $\frac{-207 + 80}{2}$  to find the average of the temperature extremes on Mars.

39. **ANALYZE GRAPHS** The *mean* of a set of data is the sum of the data divided by the number of items in the data set. The graph shows the approximate depths where certain fish are found in the Caribbean. What is the mean depth of the fish shown?



### EXTRA PRACTICE

See pages 673, 705.





### H.O.T. Problems

40. **OPEN ENDED** Write a division sentence with a quotient of  $-12$ .
41. **Which One Doesn't Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.
- $-66 \div 11$

$-32 \div (-4)$

$16 \div (-4)$

$-48 \div 4$
42. **PATTERNS** Find the next two numbers in the pattern  $729, -243, 81, -27, 9, \dots$ . Explain your reasoning.
43. **CHALLENGE** Order from least to greatest all of the numbers by which  $-20$  is divisible.
44. **WRITING IN MATH** Evaluate  $-2 \cdot (2^2 + 2) \div 2^2$ . Justify each step in the process.

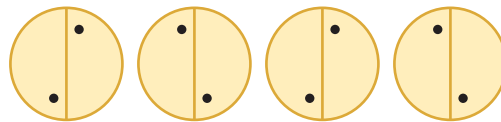


### TEST PRACTICE

45. Find  $18 \div (-3)$ .
- A  $-6$   
 B  $\frac{-1}{6}$   
 C  $6$   
 D  $15$
46. On December 24, 1924, the temperature in Fairfield, Montana, fell from  $63^\circ\text{F}$  at noon to  $-21^\circ\text{F}$  at midnight. What was the average temperature change per hour?
- F  $-3.5^\circ\text{F}$   
 G  $-7^\circ\text{F}$   
 H  $-42^\circ\text{F}$   
 J  $-84^\circ\text{F}$

### Spiral Review

47. **GEOMETRY** What is the next figure in the pattern shown at the right? (Lesson 2-7)



**Multiply.** (Lesson 2-6)

48.  $14(-2)$       49.  $-20(-3)$       50.  $-5(7)$       51.  $(-9)^2$

52. Find  $6 - (-12)$ . (Lesson 2-5)

53. **DIVING** Valentina jumped into 10 feet of water and touched the bottom of the pool before she surfaced. Write an integer to describe where Valentina was in relation to the surface of the water when she touched the bottom of the pool. (Lesson 2-1)

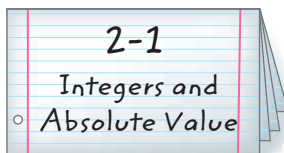
**Find each square root.** (Lesson 1-3)

54.  $\sqrt{324}$       55.  $\sqrt{900}$       56.  $\sqrt{196}$

**FOLDABLES**  
 Study Organizer

**GET READY** to Study

Be sure the following Big Ideas are noted in your Foldable.


**BIG Ideas**
**Absolute Value** (Lesson 2-1)

- The absolute value of a number is the distance the number is from zero on a number line.

**Comparing and Ordering Integers** (Lesson 2-2)

- When two numbers are graphed on a number line, the number to the left is always less than the number to the right.

**Graphing Points** (Lesson 2-3)

- On a coordinate plane, the horizontal number line is the  $x$ -axis and the vertical number line is the  $y$ -axis. The origin is at  $(0, 0)$  and is the point where the number lines intersect. The  $x$ -axis and  $y$ -axis separate the plane into four quadrants.

**Integer Operations** (Lessons 2-4, 2-5, 2-6, 2-8)

- To add integers with the same sign, add their absolute value. The sum is positive if both integers are positive and negative if both integers are negative.
- The sum of any number and its additive inverse is 0.
- To add integers with different signs, subtract their absolute values. The sum is positive if the positive integer's absolute value is greater and negative if the negative integer's absolute value is greater.
- To subtract an integer, add its opposite.
- The product or quotient of two integers with different signs is negative.
- The product or quotient of two integers with the same sign is positive.

**Key Vocabulary**

<b>absolute value</b> (p. 81)	<b>origin</b> (p. 88)
<b>additive inverse</b> (p. 96)	<b>positive integer</b> (p. 80)
<b>coordinate plane</b> (p. 88)	<b>quadrant</b> (p. 88)
<b>graph</b> (p. 80)	<b><math>x</math>-axis</b> (p. 88)
<b>integer</b> (p. 80)	<b><math>x</math>-coordinate</b> (p. 88)
<b>negative integer</b> (p. 80)	<b><math>y</math>-axis</b> (p. 88)
<b>opposites</b> (p. 96)	<b><math>y</math>-coordinate</b> (p. 88)
<b>ordered pair</b> (p. 88)	

**Vocabulary Check**

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. Integers less than zero are positive integers.
2. The origin is the point where the  $x$ -axis and  $y$ -axis intersect.
3. The absolute value of 7 is  $-7$ .
4. The sum of two negative integers is positive.
5. The  $x$ -coordinate of the ordered pair  $(2, -3)$  is  $-3$ .
6. Two integers that are opposites are also called additive inverses.
7. The product of a positive and a negative integer is negative.
8. The  $x$ -axis and the  $y$ -axis separate the plane into four coordinates.
9. The quotient of two negative integers is negative.

## Lesson-by-Lesson Review

## 2-1

## Integers and Absolute Value (pp. 80–83)

Write an integer for each situation.

10. a loss of \$150
11. 350 feet above sea level
12. a gain of 8 yards
13.  $12^{\circ}\text{F}$  below 0

Evaluate each expression.

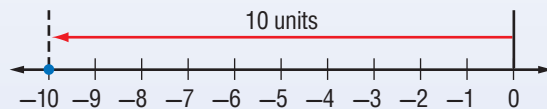
14.  $|100|$
15.  $|-32|$
16.  $|-16| + |9|$
17. **JUICE** Mavis drank 48 milliliters of apple juice before replacing the carton in the refrigerator. Write an integer that shows the change in the volume of juice in the carton.

**Example 1** Write an integer for 8 feet below sea level.

Since this situation represents an elevation *below* sea level,  $-8$  represents the situation.

**Example 2** Evaluate  $|-10|$ .

On the number line, the graph of  $-10$  is 10 units from 0.



So,  $|-10| = 10$ .

## 2-2

## Comparing and Ordering Integers (pp. 84–87)

Replace each  $\bullet$  with  $<$  or  $>$  to make a true sentence.

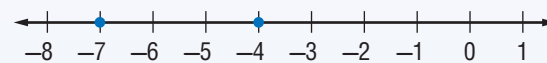
18.  $-3 \bullet -9$
19.  $8 \bullet -12$
20.  $-3 \bullet 3$
21.  $|-10| \bullet |-13|$
22.  $25 \bullet |8|$
23.  $0 \bullet |-4|$

Order each set of integers from least to greatest.

24.  $\{-3, 8, -10, 0, 5, -12, 9\}$
25.  $\{-21, 19, -23, 14, -32, 25\}$
26.  $\{-17, -18, 18, 15, -16, 16\}$
27. **WEATHER** The high temperatures in degrees Celsius for ten cities were 0, 10,  $-5$ , 12, 25,  $-6$ , 20,  $-10$ , 5 and 2. Order these temperatures from least to greatest.

**Example 3** Replace  $\bullet$  with  $<$  or  $>$  to make  $-4 \bullet -7$  a true sentence.

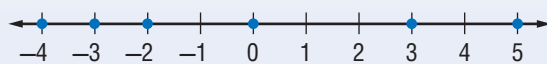
Graph each integer on a number line.



Since  $-4$  is to the right of  $-7$ ,  $-4 > -7$ .

**Example 4** Order the integers  $-4, -3, 5, 3, 0, -2$  from least to greatest.

Graph the integers on a number line.



Order the integers by reading from left to right:  $-4, -3, -2, 0, 3, 5$ .

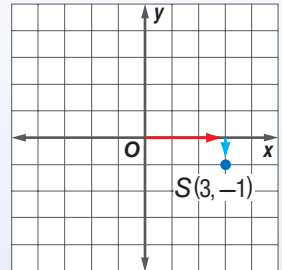
**2-3 The Coordinate Plane** (pp. 88–92)

On graph paper, draw a coordinate plane. Then graph and label each point.

28.  $E(1, -4)$
29.  $F(-4, 2)$
30.  $G(-2, -3)$
31.  $H(4, 0)$
32. **ROUTES** Starting at the school, Pilar walked 1 block east and 3 blocks south. From there, she walked 5 blocks west and 4 blocks north to the park. If the school represents the origin, what is the ordered pair for the park?

**Example 5** Graph and label the point  $S(3, -1)$ .

Draw a coordinate plane. Move 3 units to the right. Then move 1 unit down. Draw a dot and label it  $S(3, -1)$ .

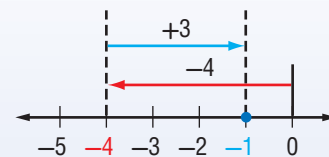


**2-4 Adding Integers** (pp. 95–99)

Add.

33.  $-6 + 8$
34.  $-4 + (-9)$
35.  $7 + (-12)$
36.  $-18 + 18$
37. **HIKING** Samuel hiked 75 feet up a mountain. He then hiked 22 feet higher. Then, he descended 8 feet, and finally climbed up another 34 feet. What is Samuel's final elevation?

**Example 6** Find  $-4 + 3$ .



So,  $-4 + 3 = -1$ .

**2-5 Subtracting Integers** (pp. 103–106)

Subtract.

38.  $-5 - 8$
39.  $3 - 6$
40.  $5 - (-2)$
41.  $-4 - (-8)$
42. **GOLF** Owen shot 2 under par while his friend Nathan shot 3 above par. By how many shots was Owen's score better than Nathan's?

**Example 7** Find  $-3 - 9$ .

$$\begin{aligned} -3 - 9 &= -3 + (-9) && \text{To subtract 9, add } -9. \\ &= -12 && \text{Simplify.} \end{aligned}$$

## 2-6

**Multiplying Integers** (pp. 107–111)

Multiply.

43.  $-4(3)$

44.  $8(-6)$

45.  $-5(-7)$

46.  $-2(40)$

**ALGEBRA** Evaluate each expression if  $a = -4$ ,  $b = -7$ , and  $c = 5$ .

47.  $ab$

48.  $-3c$

49.  $bc$

50.  $abc$

**Example 8** Find  $-4(3)$ .

$$-4(3) = -12$$

The integers have different signs.  
The product is negative.

**Example 9** Evaluate  $xyz$  if  $x = -6$ ,  $y = 11$ , and  $z = -10$ . $xyz$ 

$$= (-6)(11)(-10) \quad x = -6, y = 11, z = -10.$$

$$= (-66)(-10) \quad \text{Multiply } -6 \text{ and } 11.$$

$$= 660 \quad \text{Multiply } -66 \text{ and } -10.$$

## 2-7

**PSI: Look for a Pattern** (pp. 112–113)

Solve. Look for a pattern.

51. **HEALTH** The average person blinks 12 times per minute. At this rate, how many times does the average person blink in one day?52. **SALARY** Suki gets a job that pays \$31,000 per year. She is promised a \$2,200 raise each year. At this rate, what will her salary be in 7 years?53. **DOGS** A kennel determined that they need 144 feet of fencing to board 2 dogs, 216 feet to board 3 dogs, and 288 feet to board 4 dogs. If this pattern continues, how many feet of fencing is needed to board 8 dogs?**Example 10** A theater has 18 seats in the first row, 24 seats in the second row, 30 seats in the third row, and so on. If this pattern continues, how many seats are in the sixth row?

Begin with 18 seats and add 6 seats for each additional row.

So, there are 48 seats in the sixth row.

Row	Number of Seats
1	18
2	24
3	30
4	36
5	42
6	48

## 2-8

**Dividing Integers** (pp. 114–118)

Divide.

54.  $-45 \div (-9)$

55.  $36 \div (-12)$

56.  $-12 \div 6$

57.  $-81 \div (-9)$

**Example 11** Find  $-72 \div (-9)$ .

$$-72 \div (-9) = 8$$

The integers have the same sign. The quotient is positive.

1. **WEATHER** Adam is recording the change in the outside air temperature for a science project. At 8:00 A.M., the high temperature was 42°F. By noon, the outside temperature had fallen 11°F. By mid-afternoon, the outside air temperature had fallen 12°F and by evening, it had fallen an additional 5°F. Write an integer that describes the final change in temperature.

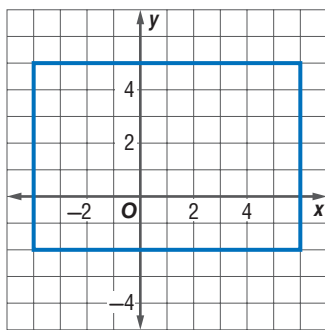
Evaluate each expression.

2.  $|-3|$                       3.  $|-18| - |6|$

Replace each ● with  $<$ ,  $>$ , or  $=$  to make a true sentence.

4.  $-3 \bullet -9$                       5.  $|9| \bullet |-12|$   
 6. The Iowa Hawkeyes recorded the following yardage in six plays: 9, -2, 5, 0, 12, and -7. Order these integers from least to greatest.

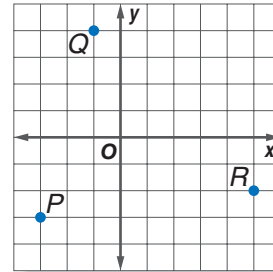
7. **MULTIPLE CHOICE** Which of the following coordinates lie within the rectangle graphed below?



- A (5, 6)                      C (-5, 1)  
 B (0, -3)                      D (-3, 0)

8. **DEBT** Amanda owes her brother \$24. If she plans to pay him back an equal amount from her piggy bank each day for six days, describe the change in the amount of money in her piggy bank each day.

Write the ordered pair for each point graphed. Then name the quadrant in which each point is located.



9. P                      10. Q                      11. R

Add, subtract, multiply, or divide.

12.  $12 + (-9)$                       13.  $-3 - 4$   
 14.  $-7 - (-20)$                       15.  $-7(-3)$   
 16.  $5(-11)$                       17.  $-36 \div (-9)$   
 18.  $-15 + (-7)$                       19.  $8 + (-6) + (-4)$   
 20.  $-9 - 7$                       21.  $-13 + 7$

22. **MULTIPLE CHOICE** Kendrick created a 6-week schedule for practicing the piano. If the pattern continues, how many hours will he practice during the sixth week?

The table shows the number of hours he practiced in the first three weeks.

Week	1	2	3
Hours	4	7	10

- F 15 hours                      H 19 hours  
 G 18 hours                      J 22 hours

Evaluate each expression if  $a = -5$ ,  $b = 4$ , and  $c = -12$ .

23.  $ac \div b$                       24.  $\frac{a-b}{3}$

25. **STOCKS** The value of a stock decreased \$4 each week for a period of six weeks. Describe the change in the value of the stock at the end of the six-week period.

## PART 1 Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- The daily low temperatures for Cleveland, Ohio, over the last five days were  $15^{\circ}\text{F}$ ,  $-2^{\circ}\text{F}$ ,  $8^{\circ}\text{F}$ ,  $-6^{\circ}\text{F}$ , and  $5^{\circ}\text{F}$ . Which expression can be used to find the average daily low temperature during the last five days?
  - $(15 + 2 + 8 + 6 + 5) \div 5$
  - $15 + 2 + 8 + 6 + 5 \div 5$
  - $[15 + (-2) + 8 + (-6) + 5] \div 5$
  - $15 + (-2) + 8 + (-6) + 5 \div 5$

### TEST-TAKING TIP

**Question 1** Check every answer choice of a multiple-choice question. Each time you find an incorrect answer, cross it off so you remember that you've eliminated it.

- Marcia runs  $r$  miles on Mondays, Tuesdays, and Thursdays. She bicycles for  $b$  miles on Wednesday and Saturdays. If she rests on Fridays and Sundays, which equation represents the total number of miles  $M$  she exercises each week?
  - $M = 3r + 2b$
  - $M = r + b$
  - $M = 2r + 3b$
  - $M = 5(r + b)$
- Simplify the expression below.
 
$$3 + 6(10 - 7) - 3^2$$
  - 0
  - 12
  - 18
  - 74

- At 8 A.M., the temperature was  $13^{\circ}\text{F}$  below zero. By 1 P.M., the temperature rose  $22^{\circ}\text{F}$  and by 6 P.M. dropped  $14^{\circ}\text{F}$ . What was the temperature at 6 P.M.?
  - $5^{\circ}\text{F}$  above zero
  - $5^{\circ}\text{F}$  below zero
  - $21^{\circ}\text{F}$  above zero
  - $21^{\circ}\text{F}$  below zero
- Sue typically spends between \$175 and \$250 each month on clothes. Which of the following is the best estimate for the amount she spends in 6 months?
  - From \$600 to \$1,200
  - From \$900 to \$1,300
  - From \$1,050 to \$1,500
  - From \$1,200 to \$1,500
- On their first play, a football team gained 17 yards. On their next play, they lost 22 yards. On their third play, they gained 14 yards. Which expression represents the total number of yards gained after the third play?
  - $17 + (-22) + 14$
  - $17 + 22 + (-14)$
  - $-17 + (-22) + (-14)$
  - $-17 + 22 + 14$
- The top four runners of a race were Alicia, Kyle, Drew, and Juanita. Drew finished before Juanita. Alicia finished after both boys, but before Juanita. What information is needed to determine the order of the runners from first to fourth?
  - Did Kyle finish before or after Drew?
  - Did Alicia finish before or after Juanita?
  - Did Drew finish before or after Alicia?
  - Did Kyle finish before or after Alicia?

8. The lowest point in Japan is Hachiro-gata (elevation  $-4$  meters), and the highest point is Mount Fuji (elevation  $3,776$  meters). What is the difference in elevation between Mount Fuji and Hachiro-gata?

- F 3,780 meters  
 G 3,772 meters  
 H 3,080 meters  
 J 944 meters

9. Which of the following relationships is best represented by the data in the table?

$x$	$y$
1	36
2	72
3	108
4	144
5	180

- A Conversion of feet to inches  
 B Conversion of inches to yards  
 C Conversion of feet to yards  
 D Conversion of yards to inches
10. A storeowner has  $n$  employees and pays each employee  $\$440$  per week. If the owner also pays  $\$d$  weekly to rent the building and  $\$w$  for utilities, which equation below represents the total  $E$  of these weekly expenses?
- F  $E = 440w + n + w$   
 G  $E = 440n + dw$   
 H  $E = 440n + d + w$   
 J  $E = 440(n + d + w)$

**PART 2** Short Response/Grid In

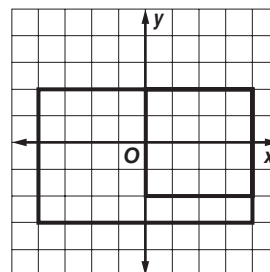
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. Nick spends a total of 75 hours per week at work and at the gym. He goes to the gym from 6:45 A.M. to 8:45 A.M., Monday through Friday. Write an equation that can be used to find  $t$ , the maximum number of hours Nick works at his job each week.
12. Find  $-8 - 17$ .

**PART 3** Extended Response

Record your answers on the answer sheet provided by your teacher or a sheet of paper. Show your work.

13. A rectangle and a square are graphed on a coordinate plane. Use the graph below to answer the questions.



- a. Pick an ordered pair both objects have in common.  
 b. Find an ordered pair that is inside the rectangle, but not the square.  
 c. How could you increase the size of the square so that it still lies within the rectangle? What are the four coordinates?  
 d. Draw a graph with the new square inside the rectangle.

**NEED EXTRA HELP?**

If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson...	1-6	1-7	1-4	2-4	1-1	2-4	1-1	2-5	2-7	1-6	1-7	2-5	2-3