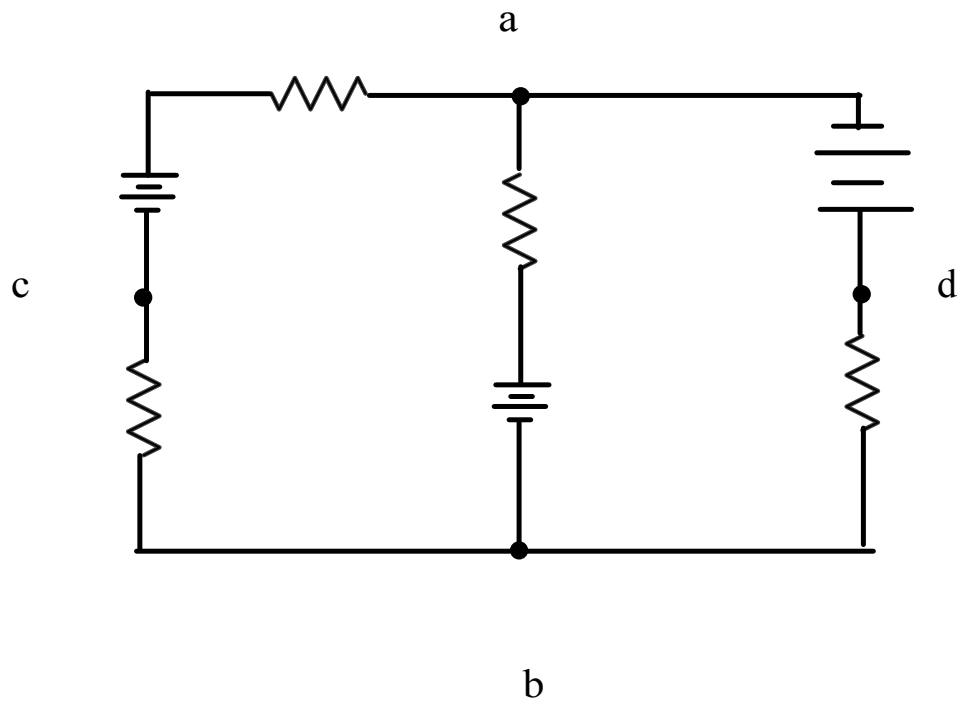
Multi-loop Circuits and Kirchoff's Rules

Junction is a point where at least three circuit paths meet.
Branch is a path connecting two junctions.

In the circuit below, there are two junctions, labeled a and b. There are three branches: these are the three paths from a to c to b and from a to d to b.

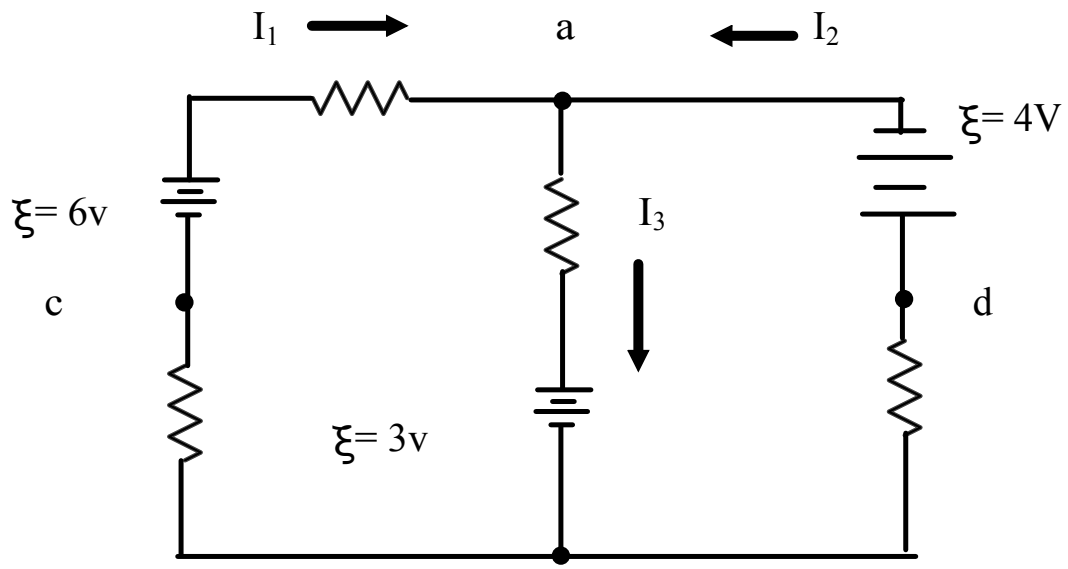


Finding the current, emf or resistance in all branches of a multi-loop circuit is done by following guidelines known as Kirchoff's rules. These guidelines also apply to very simple circuits.

Kirchoff's first rule : the junction rule. The sum of the currents coming in to a junction is equal to the sum leaving the junction. (Basically this is conservation of charge)

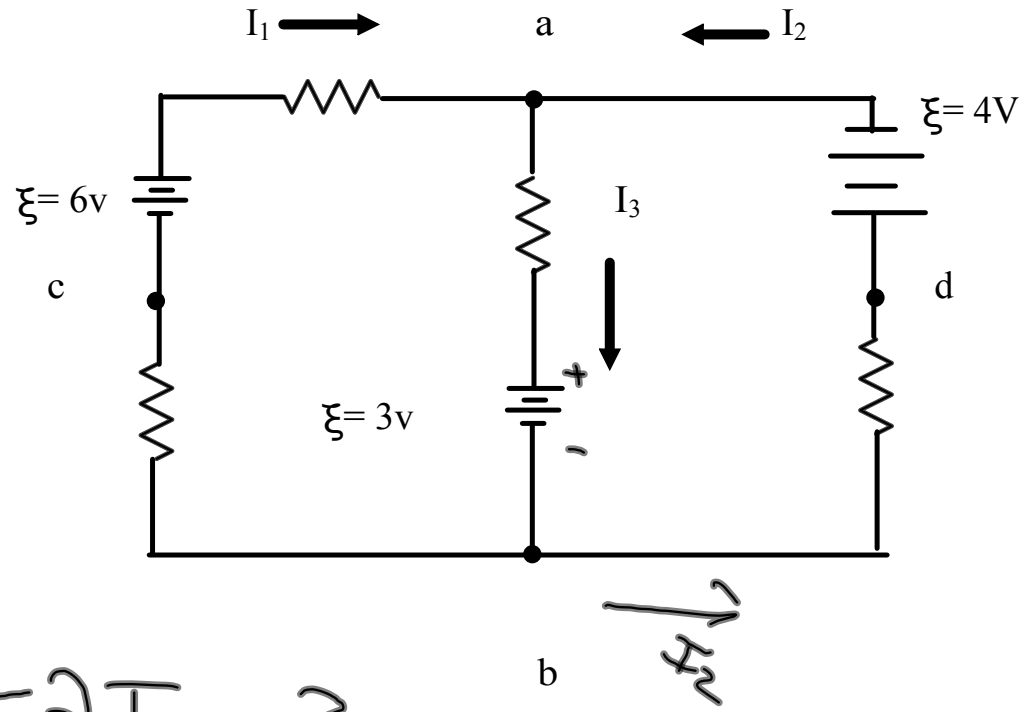
Kirchoff's second rule : the loop rule. The sum of all the potential differences around a complete loop is equal to zero. (Conservation of energy)

$I_1 + I_2 = I_3$



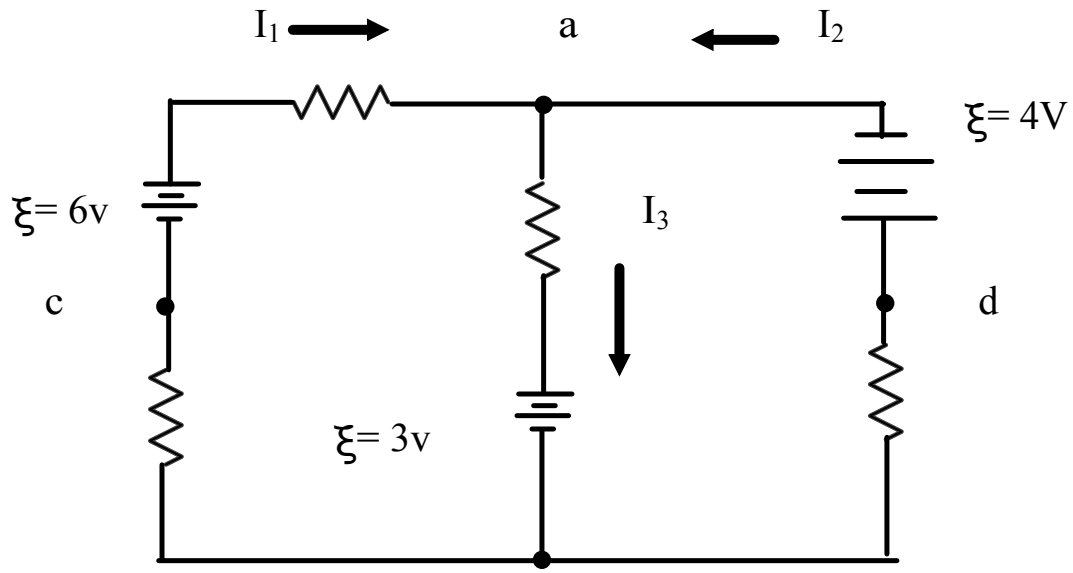
In other words, the total emf across the whole circuit when going from apoint, around the loop, back to that point is zero.

Take the above circuit into account where all the resistor have values of 2.0 ohms. Find the current through each of the loops of the circuit.



$-2I_3 - 3 - 2I_2 - 4 = 0$

$-2I_3 - 2I_2 = 7$



$-2I_3 - 3 - 2I_1 + 6 - 2I_1 = 0$

$-2I_3 - 4I_1 = -3$

$$I_1 + I_2 = I_3 \qquad I_2 = I_3 - I_1$$

$$-2I_3 - 2I_2 = 7$$

$$-2I_3 - 4I_1 = -3$$

$$-2I_3 - 2(I_3 - I_1) = 7$$

$$-2I_3 - 2I_3 + 2I_1 = 7$$

$$-4I_3 + 2I_1 = 7$$

$$2I_1 = 7 + 4I_3$$

$$I_1 = \frac{7}{2} + 2I_3$$

$$I_1 = \frac{7}{2} + 2I_3$$

$$-2I_3 - 4I_1 = -3$$

$$-2I_3 - 4\left(\frac{7}{2} + 2I_3\right) = -3$$

$$-2I_3 - 14 - 8I_3 = -3$$

$$-10I_3 = 11$$

$$I_3 = -\frac{11}{10} \text{ amp}$$